Turbomachinery – 2

SOE3211/2 Fluid Mechanics lecture 9

Real turbomachines (A)

In real turbomachines the flow is 3-d : we could write it as in cylindrical polar coordinates

$$\underline{v} = v_r \underline{\hat{r}} + v_\theta \underline{\hat{\theta}} + v_z \underline{\hat{z}}$$

where $\underline{\hat{r}}$, $\underline{\hat{\theta}}$, $\underline{\hat{z}}$ are unit vectors in radial, tangential, axial directions.

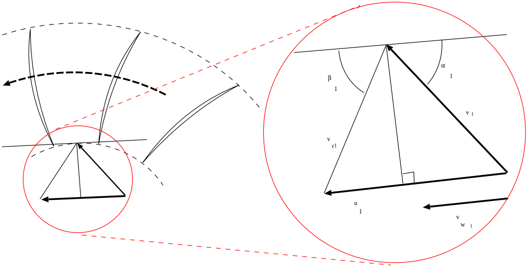
However we can make a simplifying assumption ; that $\underline{v}$ is a fn. of r only. This is equivalent to assuming :

1. the blades are infinitely thin – pressure difference across blade produces torque
2. the flow is axisymmetric (number of blades $\rightarrow \infty$)
3. no variation axially

Define the following symbols :

$$
\begin{aligned}
v &= \text{absolute velocity} \\
v_w &= \text{tangential (whirl) velocity} \\
v_f &= \text{flow velocity} \\
u &= \text{impeller velocity due to} \\
\omega &= \text{angular rotation} \\
v_r &= \text{velocity relative to impeller}
\end{aligned}
$$

$$\underline{v}_{r1} = \underline{v}_1 - \underline{u}_1$$

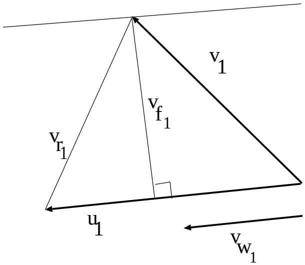


'No shock' condition (A)

This is where when the fluid enters and leaves at the angle the blade is set, i.e. $\beta' = \beta$

$\underline{v}_f$ is normal to the control surface, so relates to the flux into the C.V.

$\underline{v}_w$ is the whirl velocity at the entry to the C.V.



Momentum balance (A)

We know that applying conservation of linear momentum in the integral formulation gives the force on a body. Similarly for rotating flows

$$\underline{T}_{CV} = \iint (\rho \underline{r} \times \underline{v}) \underline{v}.d\underline{A}$$

is the net torque

We are interested in the torque in the z direction :

$$T_z = \iint_{out} (\rho r v_w) \underline{v}.d\underline{A} - \iint_{in} (\rho r v_w) \underline{v}.d\underline{A}$$

But

$$\iint \underline{v}.d\underline{A}$$

so

$$T_z = (\rho r v_w v_f A)_2 - (\rho r v_w v_f A)_1$$

However conserving mass gives the mean flow

$$\dot{m} = (\rho v_f A)_1 = (\rho v_f A)_2$$

So

$$T_z = \dot{m}\left[r_2 v_{w2} - r_1 v_{w1}\right]$$

Power = Torque × Angular Velocity, so

$$P = \dot{m}\omega\left[r_2 v_{w2} - r_1 v_{w1}\right]$$

Also $u = r\omega$ for the impeller

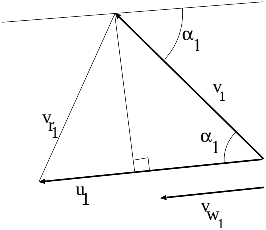
$$P = \dot{m}\left[u_2 v_{w2} - u_1 v_{w1}\right]$$

As before we want to be able to express this in terms of the head

$$H_{imp} = \frac{P}{\dot{m}g} = \frac{1}{g}\left(u_2 v_{w2} - u_1 v_{w1}\right)$$

– Euler's equation

Express this using absolute velocities



$$v_{w1} = v_1 \cos\alpha_1$$

$$v_{r1}^2 = u_1^2 + v_1^2 - 2u_1 v_1 \cos\alpha_1$$

Combining these gives

$$u_1 v_1 \cos\alpha_1 = u_1 v_{w1} = \frac{1}{2}\left(u_1^2 - v_{r1}^2 + v_1^2\right)$$

We can write similar expressions for location 2. Substituting these

$$H_{imp} = \frac{v_2^2 - v_1^2}{2g} + \frac{u_2^2 - u_1^2}{2g} + \frac{v_{r1}^2 - v_{r2}^2}{2g}$$

- $\frac{v_2^2 - v_1^2}{2g}$: increase in k.e. of the fluid in the impeller
- $\frac{u_2^2 - u_1^2}{2g}$: energy used putting fluid into circular motion about the impeller
- $\frac{v_{r1}^2 - v_{r2}^2}{2g}$: static head gained due to reduction in relative velocity as fluid goes through impeller

Centrifugal impeller (A)

Rotating with angular velocity ω

$$\Rightarrow \quad u_1 = r_1\omega$$
$$u_2 = r_2\omega$$

The mass flow $\dot{m}$ for thin blades

$$\dot{m} = \rho_1 2\pi r_1 b_1 v_{f1} = \rho_2 2\pi r_2 b_2 v_{f2}$$

If the flow is incompressible

$$r_1 b_1 v_{f1} = r_2 b_2 v_{f2}$$