Turbulent B.L. cont

SOE3211/2 Fluid Mechanics lecture 6

Skin Friction coefficient (A)

Define a *mean skin friction coefficient* for an *average* value of τ_0

$$\overline{C_f} = \frac{\overline{\tau_0}}{\frac{1}{2}\rho U_\infty^2}$$

NB. this is an average over the length of the plate (not a time average!)

Also define a Reynolds number based on the plate length L

$$\mathcal{R}e_L = \frac{U_\infty L}{\nu}$$

There are a number of formulae for $\overline{C_f}(\mathcal{R}e_L)$

By integrating the Blasius result

$$C_f = \frac{0.664}{\sqrt{\mathcal{R}e_x}}, \qquad \overline{C_f} = \frac{1.33}{\sqrt{\mathcal{R}e_L}}$$

For a turbulent BL. we have the results

$$C_f = \frac{0.0592}{\mathcal{R}e_x^{1/5}}, \qquad \overline{C_f} = \frac{0.074}{\mathcal{R}e_L^{1/5}} \qquad \mathcal{R}e < 10^7$$

Since these are based on empirical data, many different functional relations and constants are around.

Eg :

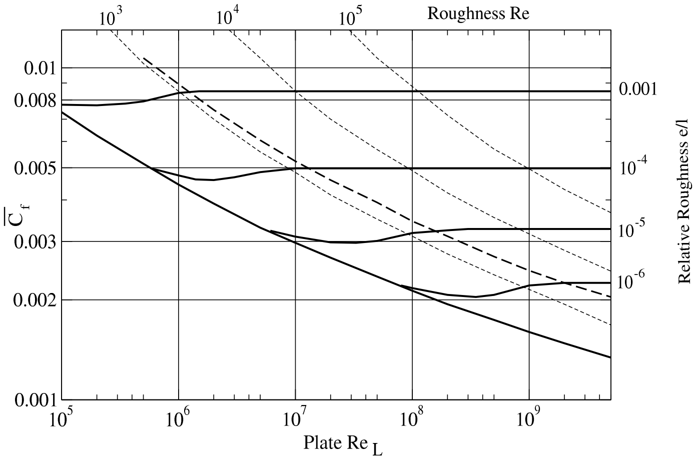
$$\overline{C_f} = 0.455 \left(\log_{10} \mathcal{R}e_L\right)^{-2.58} 10^6 < \mathcal{R}e < 10^9$$

Another important factor determining the skin friction is the roughness

If the peaks of any roughness stick through the BL, this will significantly disrupt flow over the surface – very high friction effect

If on the other hand the peaks are entirely within the laminar sublayer – minimal disruptive effect : the surface is said to be *hydraulically smooth*

Skin friction coefficient plotted against $\mathcal{R}e_L$ for various values of roughness parameter h/L.



Flow around a cylinder

Note – this is a 2-d flow. As $\mathcal{R}e$ for the flow increases, there are numerous changes in the flow patterns and thus forces on the cylinder. We will start at a low $\mathcal{R}e$ and work up.

The main force on the cylinder is the drag. Two sources :

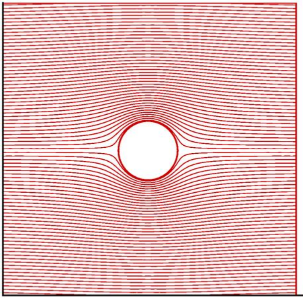
1. Pressure distribution around the cylinder – Form drag
2. Viscous forces in boundary layer – Viscous drag/Skin friction drag

Express the drag in terms of a (dimensionless) coefficient C_d :

$$C_d = \frac{F_d}{\frac{1}{2}\rho A U_0^2}$$

C_d varies with $\mathcal{R}e$: $C_d = C_d(\mathcal{R}e)$

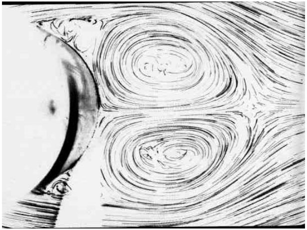
Very low $\mathcal{R}e$

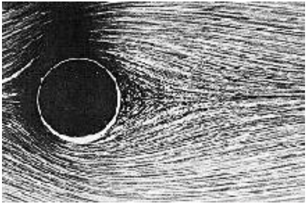


- Flow symmetric front – back $\Rightarrow$ symmetrical pressure distribution around cylinder $\Rightarrow$ form drag $\simeq 0$
- Drag forces entirely due to viscosity
- As $\mathcal{R}e$ increases, flow less symmetric

Note : $\mathcal{R}e$ are somewhat approximate – depend on roughness of cylinder, details of inlet flow, etc. etc.

$2 < \mathcal{R}e < 30$



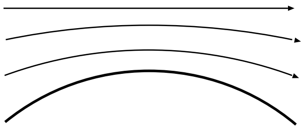


(see http://www.che.eng.ohio-state.edu/~KOELLING/81508/K_Koelling_81508_suggestions.htm)

Attached eddies form behind cylinder. Why?

Until now, we have considered *flat* boundary layers with *no* pressure gradient.

Boundary layer on *curved* surface :



Flow must speed up, then slow down (continuity). This implies there must be a pressure gradient along the surface.

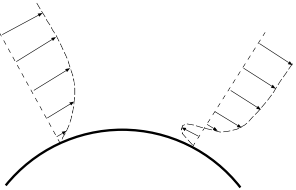
If the pressure *decreases* in the downstream direction

- the boundary layer reduces in thickness
- called a *favourable* pressure gradient

If the pressure *increases* in the downstream direction

- the boundary layer increases in thickness
- called an *adverse* pressure gradient

If the pressure gradient is sufficiently adverse, it can cause the flow to reverse in the boundary layer. This causes *recirculation* – the boundary layer is said to *separate*



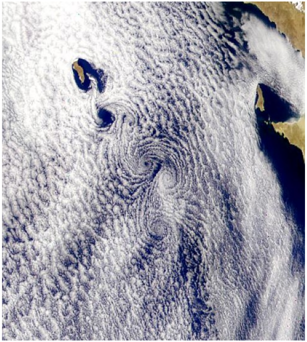
von Karman Vortex Street

As $\mathcal{R}e$ increases, the trailing vortices lengthen and start to oscillate ($30 < \mathcal{R}e < 90$).

Eventually they fall off and are carried downstream ($250 < \mathcal{R}e < 10^3$).

– the von Karman Vortex street

(see Nasa web site)



The vortices are shed from alternate sides of the cylinder. As one is shed, a new one grows on the other side. Thus, downstream is a double row of vortices being carried along in the flow

2 facts about vorticity :

1. In many flows vorticity is conserved
2. A body with associated vorticity in a flow experiences a transverse (lift) force – the Magnus effect

We will discuss the Magnus effect later (wings, aerofoils).

If the cylinder sheds a (+) vortex, it retains an opposite vorticity (–) attached – thus it experiences a force in one direction.

When the opposite vortex is shed, it carries off the (–) vorticity – the cylinder now has (+) vorticity, so experiences a force the other way.

Thus the cylinder will (try to) vibrate – Aeolean harp effect

Examples

- Singing pylons
- Unlaiden roofracks
- Tacoma narrows bridge

Look at the following :

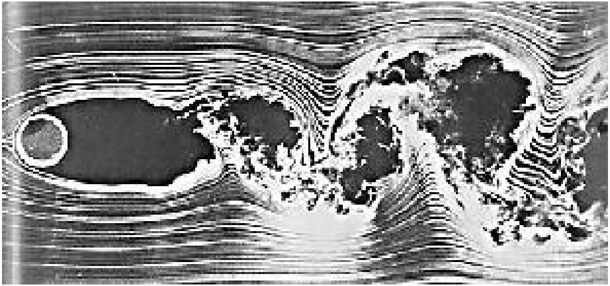
- Nasa web site – includes an animation of vortex shedding
- Tacoma Narrows bridge disaster

We can define a dimensionless number for this – the Strouhal number. If the frequency of vibration is f then

$$Str = \frac{fd}{U_0} = 0.198\left(1 - \frac{19.7}{\mathcal{R}e}\right)$$

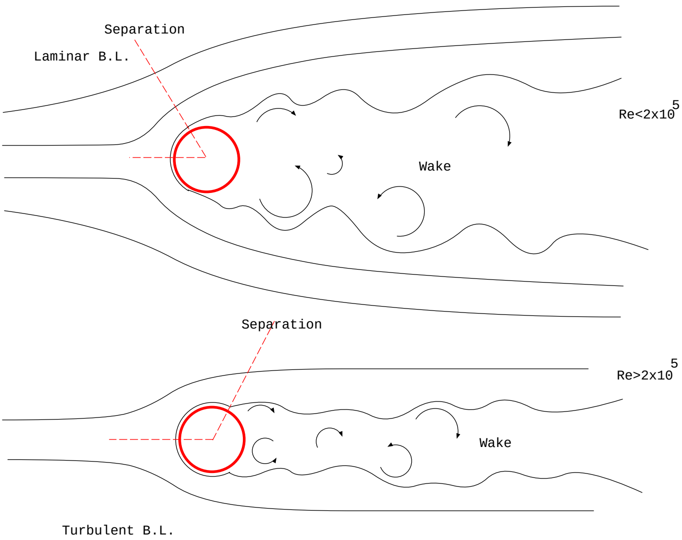
(valid for $250 < \mathcal{R}e < 2 \times 10^5$)

Turbulent wake region

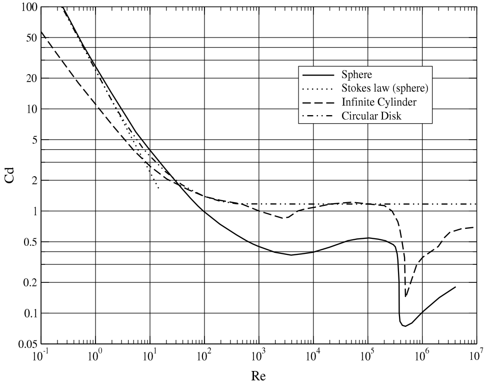


For $\mathcal{R}e > 10^3$ the von Karman vortex street degenerates into a turbulent wake. The boundary layer in front is still laminar, but separates at an angle of 81°

At $\mathcal{R}e \sim 5 \times 10^5$ the boundary layer becomes turbulent. This delays separation, (which now occurs behind the cylinder rather than in front) leading to a reduced form drag. This drop in drag is known as the *drag crisis*. F_d as well as C_d can be reduced by increasing U_0 in this region.


Separation
Laminar B.L.
Re<2x10^5
Wake
Separation
Re>2x10^5
Wake
Turbulent B.L.

We can plot $C_d(\mathcal{R}e)$ for cylinders over this range of flow conditions :



To summarise :

- $\mathcal{R}e < 0.5$: no separation, low form drag, symmetrical. C_d varies as U_0 because of the skin friction drag (drag proportional to U_0).
- $2 < \mathcal{R}e < 30$: separation occurs, attached eddies. Significant form drag because symmetry broken. C_d varies as U_0^2 since the point of separation shifts.
- $30 < \mathcal{R}e < 90$: attached eddies become unstable
- $250 < \mathcal{R}e < 10^3$: von Karman vortex street.
- $10^3 < \mathcal{R}e < 5 \times 10^5$: laminar boundary layer up to 81°, then separation. Pressure drag $\gg$ skin friction; drag coefficient pretty constant.
- $\mathcal{R}e > 5 \times 10^5$: boundary layer becomes turbulent. Separation delayed, so lower pressure drag, drag crisis.

Some other websites

- Dantec dynamics
- Virtual Album of Fluid Motion
- Some CFD-generated results.
- Pictures of cylinders in laminar flow
- Some further examples