Background (A)
Efficiency (A)
Design issues (A)
Wind resource modelling (A)
Wind statistics (B)
Blade aerodynamics (B)

Wind turbines

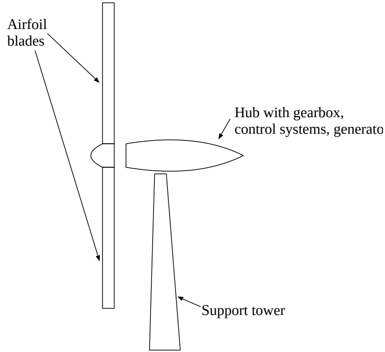
SOE3211/2 Fluid Mechanics lecture 10

Background (A)

Much interest in wind power for electricity generation – currently only really competative renewable energy source. UK has significant wind 'reserves'.

Several designs investigated over last 20 years – however industry has opted for single design : Horizontal Axis Wind Turbine (HAWT). Features :

- 2-3 airfoil blades (composite construction)
- Blades connected to hub : axis of rotation horizontal
- Hub/shaft/generation gear placed on top of support tower



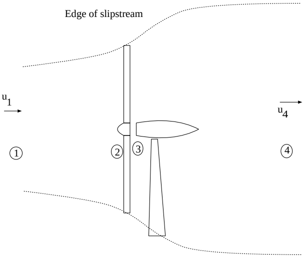
Energy generation – implies momentum extracted from airflow

$$\text{speed downstream} < \text{speed upstream}$$

– streamlines expand through rotor disk.

Analyse this through Bernoulli.

Efficiency (A)



Applying Bernoulli $1 \to 2$

$$p_a + \frac{1}{2}\rho u_1^2 = p_2 + \frac{1}{2}\rho u_2^2$$

Applying Bernoulli $3 \to 4$

$$p_3 + \frac{1}{2}\rho u_3^2 = p_a + \frac{1}{2}\rho u_4^2$$

Assume that $u_2 = u_3$, so

$$p_2 - p_3 = \frac{1}{2}\rho(u_1^2 - u_2^2)$$

Thrust on the turbine

$$F = \rho Q(u_1 - u_4) = A(p_2 - p_3)$$

Combining these we find

$$u_2 = u_3 = \frac{1}{2}(u_1 + u_4)$$

Efficiency of turbine

$$\begin{aligned}
\eta &= \frac{\text{loss of k.e. from airstream}}{\text{undisturbed power through rotor disk area}} \\
&= \frac{\frac{1}{2}\rho A u_2 (u_1^2 - u_4^2)}{\frac{1}{2}\rho A u_1 u_1^2} \\
&= \frac{(u_1 + u_4)(u_1^2 - u_4^2)}{2u_1^3} \\
&= \frac{1}{2}(1 + u_r - u_r^2 - u_r^3)
\end{aligned}$$

with $u_r = u_4/u_1$

Maximum efficiency – look for turning point

$$\frac{d\eta}{du_r} = \frac{1}{2}(1 - 2u_r - 3u_r^2) = 0$$

$$\Rightarrow u_r = \frac{1}{3}$$

for which

$$\eta = \frac{16}{27} = 59.3\%$$

– the *Betz* limit

Interpretation :

- Maximum energy extraction when $u_4 = 0$ – no flow through turbine, zero power
- If $u_4 = u_1$, flow rate maximised, but no energy extracted
- Betz represents optimum energy extraction.

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Design issues (A)

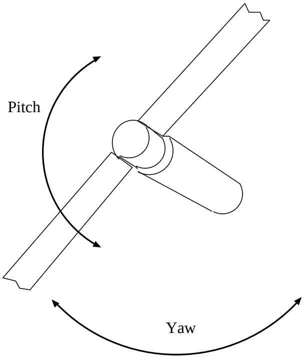
Various (fluids-related) design issues of importance, including

Vibration – air turbulence, vortex shedding, (blade rotation) all generate vibrations at specific frequencies. This can cause fatigue. Solution : design the installation so that its resonant frequencies $\neq$ any likely driving frequencies.

Noise – quite a significant NIMBY issue. Design turbines to reduce aerodynamic noise, model sound propagation to investigate environmental impact.

System control. Need turbine to point into wind, run at constant speed : also cut out if wind speed too high.

Yaw, pitch – rotation of assembly to point into prevailing wind



– also to reduce efficiency in high winds.

Also : control pitch of blades themselves to change aerodynamic forces or stall the blades when necessary.

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Wind resource modelling (A)

Detailed modelling of aerodynamics of blades etc – use CFD.

However – also interested in the longer-term average winds – prevailing wind direction, strength etc – particularly when considering wind farm siting.

Atmospheric Boundary Layer (ABL) :

- At heights $> 1\ \mathrm{km}$, wind hardly affected by presence of ground
- At ground level, $u = 0$ – thus there must be a boundary layer
- Wind speeds in ABL taken to be logarithmic

$$u(z) = \frac{u^*}{k} \log\left(\frac{z}{z_0}\right)$$

with z_0 as roughness parameter – varies from $O(\mathrm{mm})$ for smooth terrain (ice, water) to $o(\mathrm{m})$ (urban areas).

- Sometimes easier to use power law profile

$$\frac{u(z)}{u(z_r)} = \left(\frac{z}{z_r}\right)^{\alpha}$$

Non-flat terrain – mesoscale modelling (atmospheric processes on spatial scales $1 - 1000\ \mathrm{km}$) – typically use combination of ABL, simple modifications for terrain features, and wind atlas.

Examples – WAsP, MS3DHJ – extensively used in siting wind farms.

Larger scale – wind resource information

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Wind statistics (B)

Wind is intermittent and fluctuating. Characterise fluctuation in terms of a *probability density function* (pdf) :

Definition

The pdf $p(u)$ is the probability that the wind speed lies between u and $u + du$.

Using this, the probability that the wind speed is between U_a and U_b is given by

$$p(U_a \leq u \leq U_b) = \int_{U_a}^{U_b} p(u)du$$

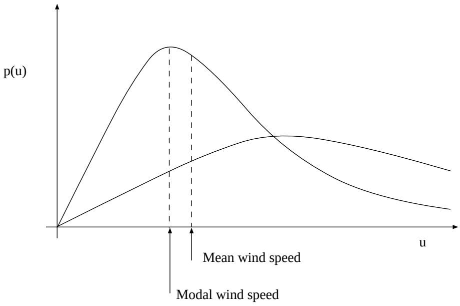
Of course

$$\int_0^{\infty} p(u)du = 1$$

Average wind power generated can be calculated if the machine power curve $P_w(u)$ is known :

$$\overline{P_w} = \int_0^\infty P_w(u)p(u)du$$

The actual shape of the pdf for wind is quite complex – often characterised by *Rayleigh* or *Weibull* distributions. Curves are not symmetrical :



Mean wind speed can be evaluated :

$$\overline{u} = \int_0^\infty u p(u) du$$

not necessarily the same as the modal wind speed (most frequent wind speed).

Fluctuations around this – turbulence intensity – also important parameter.

Available wind power dependant on u^3. Often use *root mean cube* wind speed as index of 'average' wind speed :

$$u_{rmc} = \sqrt[3]{\overline{u^3}}$$

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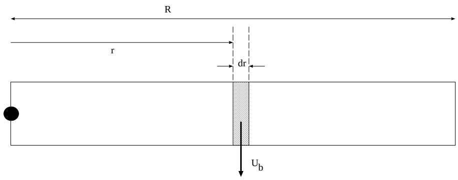
Blade aerodynamics (B)

Individual blades are airfoils with carefully-chosen profiles. Lift (and drag!) forces on the blades create a torque which drives the turbine.

Blades can turn faster than wind speed (this makes the system more efficient). Speed ratios up to 10 can be used. Force on blade $\propto u_{rel}^2$, where u_{rel} is the relative speed of the blade to the air – a combination of rotation and wind speed.

u_{rel} increases with distance from the hub (since $u_b = \Omega r$). Thus, to calculate the torque, consider small *blade element* and integrate.



Determine lift force for small element

$$dF = \frac{1}{2} C_L u_{rel}^2 b \, dr$$

and torque

$$dG = \frac{1}{2} C_L u_{rel}^2 r b \, dr$$

and integrate. Note that C_L may vary with r if necessary.