SOE3211/2 Thermofluids and Energy Conversion A/B

Fluid Dynamics

Dr Gavin Tabor/Dr Christos Makropoulos

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Course details

Course schedule

11 lectures – Mon 12pm, Fri 12pm

Tutorial – Tue 12pm

Labs – Wed 11am-1pm

Assessment

- Assessment sheets + labs = 30%
- 1 exam = 70%

Course content – Fluids

Split into **A** and **B** categories

- **A** material core – B.Eng, M.Eng
- **B** material more advanced – M.Eng alone

Assume knowledge of :

- Mathematics – partial derivatives, diff. equations
- Fluid dynamics – potential flow, Bernoulli, pipe flows, some integral methods

Text :
"Fluid Mechanics", Douglas, Gasiorek, Swaffield

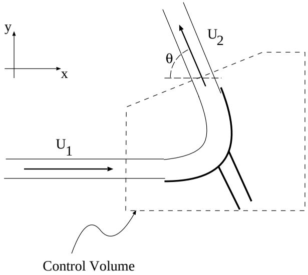
Additional 2 web lectures !!

- Dimensional Analysis (A)
- Blasius solution of b.l. flow (B)

Navier-Stokes Equations (A)

Encapsulate conservation of mass, momentum, (energy).

Used these before – e.g. force exerted on vane :



Draw a box around the flow and examine the momentum entering and leaving.

$$\begin{array}{c}\text{Change in}\\\text{momentum}\end{array} = \begin{pmatrix}\text{Momentum}\\\text{in}\end{pmatrix} - \begin{pmatrix}\text{Momentum}\\\text{out}\end{pmatrix}$$
$$= \text{Force on blade}$$

x-direction :

$$F_x = \rho A U_1^2 - (-\rho A U_1^2 \cos\theta) = \rho A U_1^2 (1 + \cos\theta)$$

y-direction :

$$F_y = \rho A U_1^2 \sin\theta$$

Often referred to as NSE in *integral form*, or *control-volume* formulation.

What happens if the velocity varies across a surface? Split surface into little pieces dA and integrate.

Continuity equation (A)

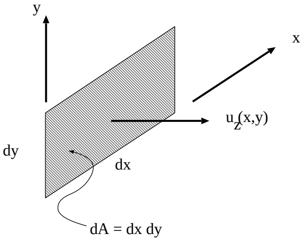
Volumetric flow rate

$$Q = A \times u$$

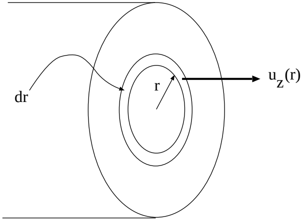
Easy if u constant – what happens if u varies

$$u = u(x, y)?$$

Need to consider small area dA. Eg cartesian coordinates :



Pipe flow – $u_z(r)$ (typically). Evaluate flux through small annulus :



Area of annulus :

$$
\begin{aligned}
dA &= \pi(r+dr)^2 - \pi r^2 \\
&= 2\pi r dr
\end{aligned}
$$

Volumetric flow through dA

$$dQ = dA \times u_z = 2\pi r dr \times u_z$$

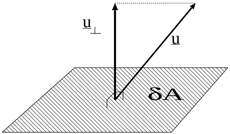
Sum this over whole area $\equiv$ integrating

$$Q = \int_0^R 2\pi r u_z(r) dr$$

Mass flux (A)

Introduce the concept of the *amount* of fluid flowing through a (possibly arbitrary) surface – a *flux* :

$$\text{Mass flow through A} = \rho u_{\perp} A = \rho \underline{u}.\underline{A}$$



$\phi^{(\rho)} = \rho \underline{u}.\underline{A}$ we call the

$\boxed{\text{Mass Flux}}$

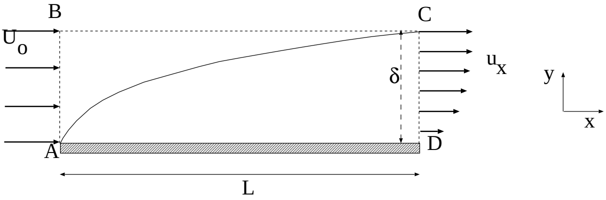
Boundary Layer Flow (A)

At a wall boundary, the velocity parallel to the wall must be zero :

$$u_{||} = 0$$

(Actually, more generally $u_{||} = V$). Away from the wall, the velocity is non-zero.

Hence there must be a region of influence of the wall, called the *boundary layer*, where the flow adapts to the presence of the wall.



Across CD,

$$\frac{u_x}{U_0} = \begin{cases} \sin\left(\frac{\pi y}{2\delta}\right) & 0 \le \frac{y}{\delta} \le 1 \\ 1 & \frac{y}{\delta} > 1 \end{cases}$$

What is the mass flux through BC?

Mass flux $\phi_{AB}^{(\rho)} = \rho U_0 (\delta \times d)$

Mass flux $\phi_{CD}^{(\rho)}$?

Mass flux $\phi^{(\rho)}_{AB} = \rho U_0(\delta \times d)$

Mass flux $\phi^{(\rho)}_{CD}$?

Small element $\delta A = \delta y \times d$ at y

Flux $\phi^{(\rho)}_{\delta A} = \rho u_x dA = \rho u_x (d \times dy)$

Mass flux $\phi^{(\rho)}_{AB} = \rho U_0(\delta \times d)$

Mass flux $\phi^{(\rho)}_{CD}$?

Small element $\delta A = \delta y \times d$ at y

Flux $\quad \phi^{(\rho)}_{\delta A} = \rho u_x dA = \rho u_x(d \times dy)$

$$
\begin{aligned}
\phi^{(\rho)}_{CD} &= \int_0^\delta \phi^{(\rho)}_{dA} \\
&= \rho U_0 d \int_0^\delta \sin\left(\frac{\pi y}{2\delta}\right) dy \\
&= \rho U_0 d \times \frac{2\delta}{\pi}
\end{aligned}
$$

Thus :

$$
\begin{aligned}
\text{Mass flux through BC} &= (\text{Flux in}) - (\text{Flux out}) \\
&= \rho U_0 \delta d \left[1 - \frac{2}{\pi} \right]
\end{aligned}
$$

Conservation of mass (A)

$$\text{Mass flux in} = \text{Mass flux out}$$

so, over any region V with faces A_i,

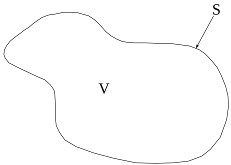
$$\sum_i \phi_{A_i}^{(\rho)} = 0 \qquad \text{– true for } \textit{incompressible} \text{ fluids}$$

More generally, mass can collect in region V. Mass of fluid density ρ :

$$\iiint_V \rho dV$$

so change of mass in V is

$$\frac{d}{dt} \iiint_V \rho dV$$



Continuity Equation (A)

$$\boxed{\frac{d}{dt}\iiint_V \rho dV + \sum_{\text{surface of V}} \text{Fluxes} = 0}$$

If the surface of V is not flat, need to sum little pieces dA, so

$$\frac{d}{dt}\iiint_V \rho dV + \iint_S \rho \underline{u}.d\underline{A} = 0$$