Turbomachinery – 2

SOE3211/2 Fluid Mechanics lecture 9

9.1 Real turbomachines (A)

In real turbomachines the flow is 3-d : we could write it as in cylindrical polar coordinates

$$\underline{v} = v_r \underline{\hat{r}} + v_\theta \underline{\hat{\theta}} + v_z \underline{\hat{z}}$$

where $\underline{\hat{r}}$, $\underline{\hat{\theta}}$, $\underline{\hat{z}}$ are unit vectors in radial, tangential, axial directions.

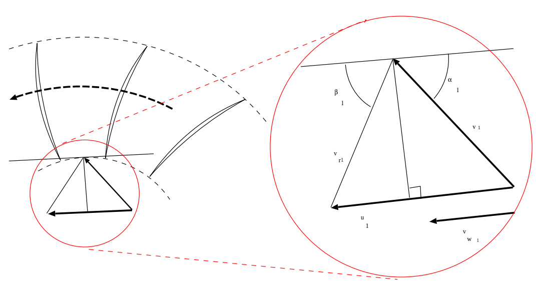
However we can make a simplifying assumption ; that $\underline{v}$ is a fn. of r only. This is equivalent to assuming :

1. the blades are infinitely thin – pressure difference across blade produces torque
2. the flow is axisymmetric (number of blades $\to \infty$)
3. no variation axially

Define the following symbols :

v	$=$	absolute velocity
v_w	$=$	tangential (whirl) velocity
v_f	$=$	flow velocity
u	$=$	impeller velocity due to
ω	$=$	angular rotation
v_r	$=$	velocity relative to impeller

$$\underline{v}_{r1} = \underline{v}_1 - \underline{u}_1$$

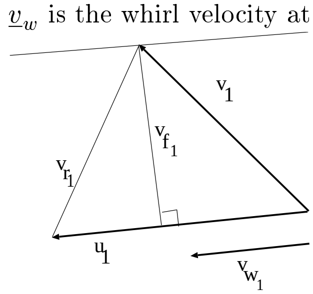


9.2 'No shock' condition (A)

This is where when the fluid enters and leaves at the angle the blade is set, i.e. $\beta' = \beta$

$\underline{v}_f$ is normal to the control surface, so relates to the flux into the C.V.

$\underline{v}_w$ is the whirl velocity at the entry to the C.V.



9.3 Momentum balance (A)

We know that applying conservation of linear momentum in the integral formulation gives the force on a body. Similarly for rotating flows

$$\underline{T}_{CV} = \iint (\rho \underline{r} \times \underline{v}) \underline{v}.d\underline{A}$$

is the net torque

We are interested in the torque in the z direction :

$$T_z = \iint_{out} (\rho r v_w) \underline{v}.d\underline{A} - \iint_{in} (\rho r v_w) \underline{v}.d\underline{A}$$

But

$$\iint \underline{v}.d\underline{A}$$

so

$$T_z = (\rho r v_w v_f A)_2 - (\rho r v_w v_f A)_1$$

However conserving mass gives the mean flow

$$\dot{m} = (\rho v_f A)_1 = (\rho v_f A)_2$$

So

$$T_z = \dot{m} \left[r_2 v_{w2} - r_1 v_{w1} \right]$$

Power = Torque $\times$ Angular Velocity, so

$$P = \dot{m} \omega \left[r_2 v_{w2} - r_1 v_{w1} \right]$$

Also $u = r\omega$ for the impeller

$$P = \dot{m}\left[u_2 v_{w2} - u_1 v_{w1}\right]$$

As before we want to be able to express this in terms of the head

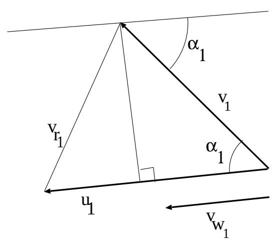
$$H_{imp} = \frac{P}{\dot{m}g} = \frac{1}{g}\left(u_2 v_{w2} - u_1 v_{w1}\right)$$

– *Euler's equation*

Express this using absolute velocities

$$v_{w1} = v_1 \cos\alpha_1$$

$$v_{r1}^2 = u_1^2 + v_1^2 - 2u_1 v_1 \cos\alpha_1$$



Combining these gives

$$u_1 v_1 \cos\alpha_1 = u_1 v_{w1} = \frac{1}{2}\left(u_1^2 - v_{r1}^2 + v_1^2\right)$$

We can write similar expressions for location 2. Substituting these

$$H_{imp} = \frac{v_2^2 - v_1^2}{2g} + \frac{u_2^2 - u_1^2}{2g} + \frac{v_{r1}^2 - v_{r2}^2}{2g}$$

$\frac{v_2^2 - v_1^2}{2g}$**:** increase in k.e. of the fluid in the impeller

$\frac{u_2^2 - u_1^2}{2g}$**:** energy used putting fluid into circular motion about the impeller

$\frac{v_{r1}^2 - v_{r2}^2}{2g}$**:** static head gained due to reduction in relative velocity as fluid goes through impeller

9.4 Centrifugal impeller (A)

Rotating with angular velocity ω

$$\Rightarrow \quad \begin{aligned} u_1 &= r_1\omega \\ u_2 &= r_2\omega \end{aligned}$$

The mass flow $\dot{m}$ for thin blades

$$\dot{m} = \rho_1 2\pi r_1 b_1 v_{f1} = \rho_2 2\pi r_2 b_2 v_{f2}$$

If the flow is incompressible

$$r_1 b_1 v_{f1} = r_2 b_2 v_{f2}$$