Differential form of NSE

Reading :

DGS sections 11.1 – 11.3, 11.7 – 11.11

Simple problems :

Q.1. Evaluate the following partial derivatives

$$\frac{\partial}{\partial x}x^3y^2, \qquad \frac{\partial}{\partial y}x^3y^2, \qquad \frac{\partial}{\partial x}(y\cos x + xyz)$$

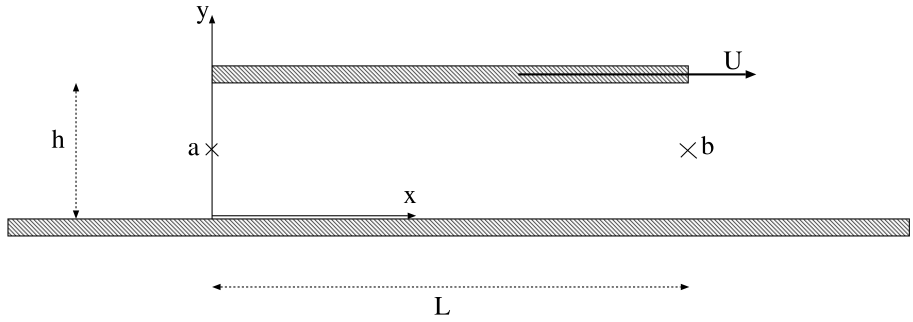
Q.2. If a 2-dimensional flow field were to have velocity components

$$u_x = U(x^3 + xy^2) \qquad \text{and} \qquad u_y = U(y^3 + yx^2)$$

would the continuity equation be satisfied?

Advanced problems :

Q.3. Flow of oil in a bearing is typically a laminar flow problem. Consider the following situation : the top bearing surface moves parallel to the bottom surface at a speed U, the gap between the two, of height h, is filled with a viscous oil.



a. Sketch how you imagine the flow will look like for this problem.

b. Write down the NSE for the problem. What are the boundary conditions?

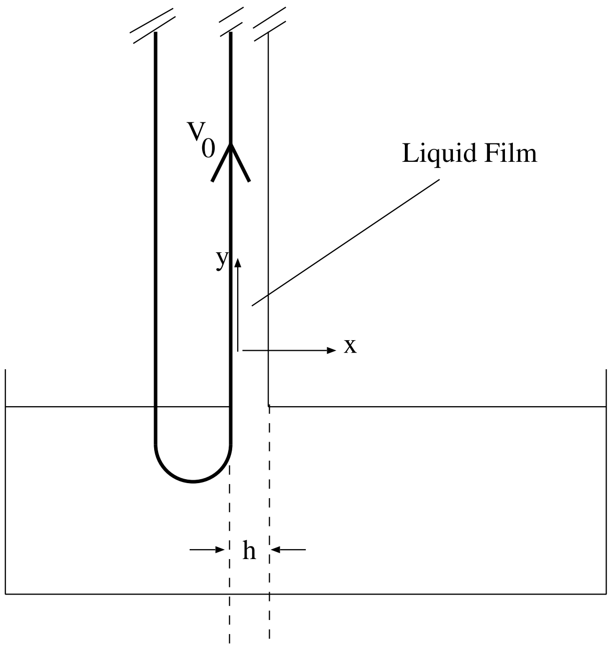
c. Show that the x-component of the fluid velocity has the following form :

$$u_x = \frac{h^2}{2\rho\nu}\left[Y^2 - Y\right]\frac{dp}{dx} + YU$$

where $Y = y/h$.

d. From this derive an expression for the flow rate Q and the pressure difference between points a and b on the diagram.

Q.4. A broad belt passes vertically upwards through a tank of liquid at constant velocity V_0 and picks up a liquid film. The effect of gravity is to make the liquid drain back into the tank, but, at a sufficiently high belt velocity it is observed that, at a short distance above the liquid surface the film thickness becomes constant and equal to h. In this region of constant film thickness it may be assumed that the flow is fully developed.



a. Assuming steady flow and negligable shear at the liquid/air interface, show that the Navier-Stokes equations reduce to

$$0 = -g + \nu \frac{d^2 u_y}{dx^2}$$

with ν the kinematic viscosity and u_y the y component of velocity (take the y axis to lie along the belt).

b. Write down the boundary conditions at the belt and at the liquid/air interface.

c. Show that the velocity profile in the developed region is given by

$$\frac{u_y}{V_0} = 1 - \frac{gh^2}{\nu V_0}\left[\frac{x}{h} - \frac{1}{2}\left(\frac{x}{h}\right)^2\right]$$

Q.5. (B) Water is flowing over a filter with a free stream velocity U in the x direction. The filter is a porous surface which draws water through at a constant velocity V in the $-y$ direction. When the apparatus is set up correctly, a boundary layer forms which is time-independent and fully developed (the flow through the filter is just enough to prevent the boundary layer from expanding). Write down the boundary conditions at the filter and a long way from the filter. Show that the NSE reduce to the form

$$u_y \frac{\partial u_x}{\partial y} = \nu \frac{\partial^2 u_x}{\partial y^2}$$

and find the solution for $u_x(y)$.

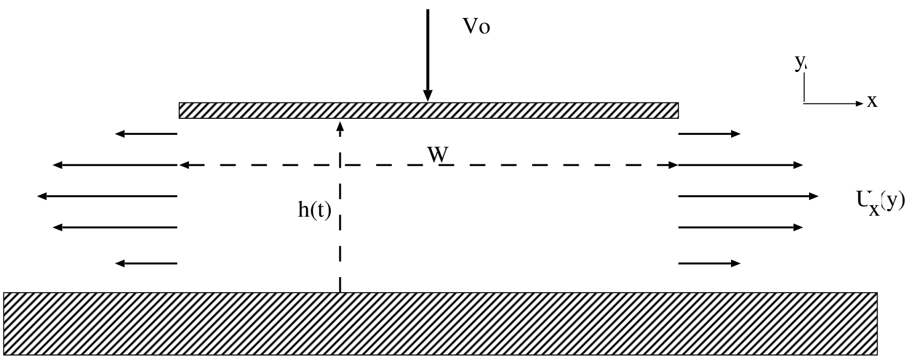
Q.6. (B) A flat plate of width W is pushed down at a speed V_0 onto a puddle of liquid on a table, causing it to spurt out at both sides. The velocity distribution of the fluid at the side is given by the formula

$$u_x(y) = 4U_0 \left[\frac{y}{h(t)} - \left(\frac{y}{h(t)} \right)^2 \right]$$

where U_0 is the maximum outflow and $h(t)$ is the height of the flat plate above the table. (You may consider the plate to be of infinite length, i.e. ignore end effects).

a. Determine $h(t)$ in terms of the velocity V_0 and the initial height h_0.

b. Calculate the maximum velocity U_0 at the exit in terms of V_0, W and $h(t)$.



(Hint : this involves the NSE in *integral* form).