SOE3152

UNIVERSITY OF EXETER

SCHOOL OF ENGINEERING AND COMPUTER SCIENCE

DEPARTMENT OF ENGINEERING

Fluid Dynamics A

Time allowed : TWO HOURS

January 2003

The marks for this module are calculated from 70% of the percentage mark for this paper plus 30% of the percentage mark for associated coursework.

Full marks may be obtained from full answers to three questions. Candidates have a free choice of questions.

This is a **closed book** examination. Candidates are permitted to use approved portable calculators. A separate formula and data sheet has been provided.

SECTION A

Question 1 (20 marks)

An incompressible Newtonian fluid of density ρ and viscosity μ flows at a steady rate along a uniform circular pipe of radius R. The stress in the fluid is given by

$$\tau(r) = \frac{r}{2}\frac{dp}{dz}$$

where r is the distance from the axis and $\frac{dp}{dz}$ is the pressure gradient along the pipe. In these coordinates, the relation between stress and strain is given by

$$\tau = \mu \frac{\partial u(r)}{\partial r}$$

1(a) (6 marks) Assuming laminar flow, show that the velocity in the pipe is given by

$$u(r) = -\frac{dp}{dz}\frac{1}{4\mu}\left(R^2 - r^2\right)$$

1(b) (6 marks) Derive an expression for the volumetric flow rate Q in terms of R, μ and the pressure gradient.

1(c) (4 marks) Use the result from part **1(b)** to show that the velocity distribution can be written as

$$u(r) = 2\overline{u}\frac{R^2 - r^2}{R^2}$$

where $\overline{u}$ is the mean speed $Q/\pi R^2$.

1(d) (4 marks) The flow velocity in the pipe is now increased until a Reynolds number of $\mathcal{R}e = 8000$ is reached. What changes in the flow and in the flow profile $u(r)$ would you expect to see in this case?

Question 2 (20 marks)

2(a) (5 marks) Sketch the energy spectrum for isotropic homogeneous turbulence.

2(b) (4 marks) Discuss the phenomenon of the Von Karman Vortex Street, indicating the significance of the Strouhal number.

2(c) (8 marks) Estimate the terminal velocity of a steel sphere of diameter d falling through water at 20° C, when

- **i.** $d = 4$ mm
- **ii.** $d = 1.2$ cm

Take the density of steel to be 7800 kg/m^3.

2(d) (3 marks) Why does a golf ball have dimples?

Question 3 (20 marks)

3(a) (4 marks) What is meant by the following terms :

i. no-shock condition

ii. hydraulic losses

A centrifugal turbine with inside diameter (ID) 40 cm and outside diameter (OD) 1 m rotates at 200 RPM and discharges 0.8 m^3/s of water. The blade height is constant at 10 cm, and the water inlet imparts a swirl velocity v_{w1} of 1.45 m/s.

3(b) (4 marks) Draw a vector triangle for the inlet velocities, and calculate the relative and absolute inlet velocities.

3(c) (4 marks) At what angle should the blades be set for the no-shock condition to hold at the inlet?

3(d) (4 marks) If the blades sweep around to an angle $\beta_2 = 40°$, what are the flow velocities (absolute and relative) at the outlet?

3(e) (4 marks) Calculate the theoretical head H_{imp}.

SECTION B

Question 4 (20 marks)

4(a) (5 marks) An aerofoil *NACA3153* has a lift coefficient that varies as

$$C_L(\alpha) = 0.6\,(1 + 0.233\,\alpha) \qquad (\alpha \text{ in degrees})$$

for $\alpha < 7.5°$. Plot this curve, and sketch how you would expect it to continue for $\alpha > 7.5°$. Explain your reasoning.

4(b) (8 marks) A helicopter has 4 blades each of length R, width c, and with a lift coefficient $C_L(r)$. Show that the vertical force developed by the rotor is given by

$$F = \frac{1}{2}q \int_0^1 C_L y^2 dy, \qquad \text{where} \quad y = \frac{r}{R}$$

and find an expression for the coefficient q in terms of the angular speed of rotation Ω.

4(c) (7 marks) A light helicopter has a rotor with blades 6 m in length and 12 cm in width. Its blades use the *NACA3153* aerofoil, and are twisted so that the angle of attack varies according to

$$\alpha(r) = \alpha_0 \left(1 - \frac{r}{2R}\right), \qquad \alpha_0 = 5°$$

If the loaded mass of the helicopter is 780 kg, at what speed must the blades rotate when the helicopter is hovering?

Question 5 (20 marks)

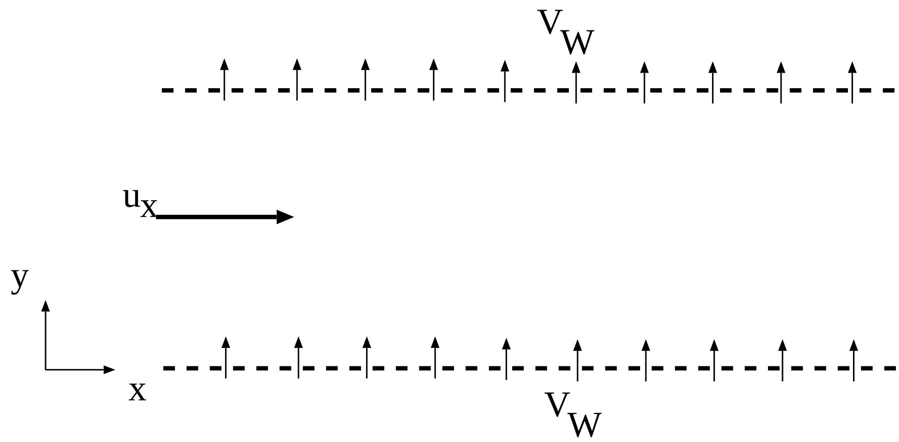
5(a) (6 marks) Given the velocity

$$\underline{u} = (Ax \sin \omega t)\underline{i} - (Ay \sin \omega t)\underline{j}$$

i. Evaluate the divergence of velocity $\nabla.\underline{u}$.

ii. Could $\underline{u}$ represent the fluid velocity for an incompressible flow? Justify your answer.

5(b) (14 marks) Water is flowing through a channel of width h as shown in the figure below. The flow is driven by a pressure gradient in the x direction $\frac{\partial p}{\partial x} = -K$, and the channel walls are porous, producing a flow $u_y = V_W$. The flow is fully developed.



i. Using the continuity equation, find an expression for the velocity u_y.

ii. Show that the velocity $u_x(y)$ satisfies the equation

$$\frac{d^2u_x}{dy^2} - \frac{V_W}{\nu}\frac{du_x}{dy} = -\frac{K}{\nu\rho}$$

iii. By considering the boundary conditions, and writing $X = \frac{du_x}{dy}$, find an expression for the velocity profile $u_x(y)$. You may assume that

$$\int \frac{dX}{aX+b} = \frac{1}{a}\log(aX+b)$$

Question 6 (20 marks)

6(a) (10 marks) The head H developed across a centrifugal pump can be shown to depend on the rotational speed of the pump N (in RPM), the diameter of the impeller D, the volumetric flow rate Q, the dynamic viscosity of the fluid μ and the density of the fluid ρ. For convenience this head is sometimes expressed as the quantity gH, where g is the accelleration due to gravity.

i. Find an dimensionless expression for the quantity gH, using as repeating variables N, D and ρ.

ii. Using Buckingham-Π theorem, find an expression for the relationship between the head H, the volumetric flow rate Q and the viscosity of the fluid μ.

iii. A more sophisticated analysis shows that the head developed also depends on the bulk modulus of the fluid K (units Pa) and the roughness η. Indicate how this changes the result from part **ii**.

6(b) (10 marks) The motion of any incompressible fluid flowing in a gravitational field can be described by the Navier-Stokes equations. Using a characteristic length scale of the flow, L_0, and a characteristic velocity of the flow, U_0, derive a dimensionless version of the momentum equation, indicating the significance of any new terms that arise from the analysis.