SOE3153 Referred/Deferred

UNIVERSITY OF EXETER

SCHOOL OF ENGINEERING AND COMPUTER SCIENCE

DEPARTMENT OF ENGINEERING

Fluid Dynamics B

Time allowed : TWO HOURS

September 2003

Full marks may be obtained from full answers to three questions. Candidates are required to answer **at least two questions from section B**. For **DEFERRED** candidates, the marks for this module are calculated from the better of : 70% of the percentage mark for this paper plus 30% of the percentage mark for associated coursework, or 100% of the percentage mark for this paper.

For **REFERRED** candidates, the marks for this module are calculated from 100% of the mark for this paper. The maximum mark that can be achieved is 40%.

This is a **closed book** examination. Candidates are permitted to use approved portable calculators. A separate formula and data sheet has been provided. Graph paper will be provided on request.

SECTION A

Question 1 (20 marks)

1(a) (6 marks) The volumetric flow Q of fluid through a circular orifice depends on the density of the fluid ρ, the area of the orifice A and the pressure drop across the orifice Δp. Using dimensional analysis, derive an expression for Q.

1(b) (6 marks) The characteristic curve for a centrifugal pump is given by the manufacturer as

$$H = 40 - 140Q - 4200Q^2$$

with the pump operating at a speed of 800 RPM. Obtain an expression for the pump characteristic for the same pump operating at 75% of its normal operating speed. Plot both sets of characteristics for Q values up to 0.05 m^3/s.

1(c) (8 marks) Running at full speed, the pump is used to pump liquid along a pipe of length 100 m and diameter 16 cm, and discharges from the end of the pipe at atmospheric pressure at an elevation of 10 m above the open feed tank. Determine the duty point for the system. (Assume a friction factor of 0.02 for the pipe and neglect entrance and exit losses).

Question 2 (20 marks)

2(a) (5 marks) Sketch the energy spectrum for homogeneous isotropic turbulence.

2(b) (5 marks) The time average $\overline{a}$ of a fluctuating quantity $a(t)$ is given by :

$$\overline{a} = \frac{1}{\Delta t}\int_0^{\Delta t} a(t)dt$$

If a and b are fluctuating quantities, show that

i. $\overline{a+b} = \overline{a} + \overline{b}$

ii. $\overline{ab} = \overline{a}\,\overline{b} + \overline{a'b'}$

where $'$ denotes the fluctuations around the time average.

2(c) (10 marks) Discuss the important consequences of turbulence in engineering. Your answer should include at least two illustrative examples.

Question 3 (20 marks)

3(a) (7 marks) Discuss the mechanisms by which an airfoil generates lift. Your answer should include mention of the following :

i. Stall.

ii. The effect of wing flaps.

3(b) (2 marks) Data for the Lissamann 7769 Airfoil is included below. Plot the polar diagram for this airfoil.

α	C_L	C_D	α	C_L	C_D
-5.00	-0.22	0.0391	11.00	1.33	0.0409
-4.00	-0.14	0.0355	12.00	1.40	0.0460
-3.00	-0.05	0.0328	13.00	1.46	0.0509
-2.00	0.05	0.0113	14.00	1.50	0.0573
-1.00	0.16	0.0114	15.00	1.52	0.0646
0.00	0.28	0.0113	16.00	1.52	0.0736
1.00	0.36	0.0140	17.00	1.50	0.0831
2.00	0.46	0.0155	18.00	1.46	0.0973
3.00	0.56	0.0166	19.00	1.38	0.1258
4.00	0.67	0.0180	20.00	1.29	0.1592
5.00	0.77	0.0182	21.00	1.22	0.1834
6.00	0.86	0.0203	22.00	1.15	0.2049
7.00	0.96	0.0226	23.00	1.10	0.2159
8.00	1.05	0.0253	24.00	1.03	0.2388
9.00	1.14	0.0283	25.00	0.96	0.2633
10.00	1.25	0.0310			

3(c) (3 marks) The Gossamer Albatross was the first human-powered aircraft to cross the English Channel. It had a wing of constant cross-section based on the Lissaman 7769 airfoil, of total width (wingtip to wingtip) 28 m,chord length 1.8 m, and a thin cockpit 2.5 m high and 3 m long containing the pilot. The total mass of the craft was 97.5 kg. The wings were set at an angle of 10°. How fast did the craft have to move in level flight to remain airborne?

3(d) (6 marks) What fraction of the drag is accounted for by the fusilage if construction gives imperfections of 3 mm height? If the fusilage can be made hydraulically smooth, what does this value drop to?

3(e) (2 marks) How much energy did the pilot expend crossing the Channel (distance 36km)?

SECTION B

Question 4 (20 marks)

4(a) (5 marks) An incompressible fluid of kinematic viscosity ν is at rest above an infinite horizontal plate $y = 0$, also at rest. At time $t = 0$ this plate starts to move in the x-direction at speed V. Show that the subsequent flow can be described by a velocity $u_x(y, t)$ which satisfies the equation

$$\frac{\partial u_x}{\partial t} = \nu \frac{\partial^2 u_x}{\partial y^2} \tag{1}$$

4(b) (8 marks) To solve equation (1) we will assume that the solution can be written in the form

$$u_x = V f(\eta) \qquad \text{where} \quad \eta = \frac{y}{\sqrt{\nu t}}$$

Given this assumption, show that $\frac{\partial u_x}{\partial t}$ can be written as

$$\frac{\partial u_x}{\partial t} = -\frac{\eta V}{2t} \frac{df}{d\eta}$$

and find a similar expression for $\frac{\partial^2 u_x}{\partial y^2}$. By substituting these results into equation (1) show that

$$\frac{df}{d\eta} = A\mathrm{e}^{-\eta^2/4}$$

where A is a constant.

4(c) (7 marks) Find an expression for the vorticity ζ of the flow, and show that at a given distance y it reaches a maximum value when $t = y^2/2\nu$.

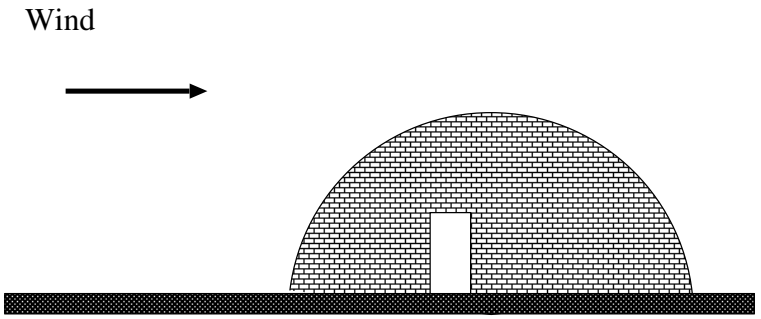
Question 5 (20 marks)

5(a) (6 marks) The potential function for crossflow around a cylinder of radius a is

$$\phi = U_0 x \left(1 + \frac{a^2}{r^2}\right)$$

Derive expressions for the velocity components u_r and u_θ from this expression, and evaluate their values at the surface of the cylinder.

5(b) (3 marks) An Antarctic base consists of a number of semicircular huts, as shown in the figure below.



How does the pressure vary around the surface of the hut?

5(c) (7 marks) Using the result from part b. determine the wind velocity at which the hut is likely to lift from the ground.

[You may assume without proof that $\int (\sin^3\theta)\, d\theta = -\cos\theta + \frac{1}{3}\cos^3\theta$.]

5(d) (4 marks) The value of u_θ at the surface of the hut, as evaluated in part a. is non-zero. Discuss the validity of this for a real fluid.

Question 6 (20 marks)

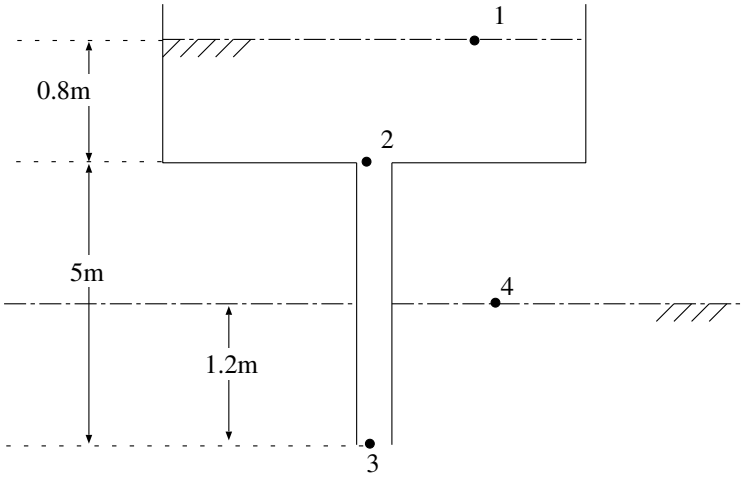
6(a) (4 marks) Inviscid fluid of constant density ρ is flowing with a steady velocity

$$\underline{u} = Axy\underline{i} + (y^2 + Bx^2)\underline{j}$$

Find the constants A and B for which this flow satisfies the continuity equation.

6(b) (6 marks) Discuss the factors that influence the frictional losses experienced by a liquid flowing through a network of pipes.

6(c) (4 marks) Water is draining from one tank through a smooth pipe of 6 cm diameter into a second tank, as shown in the figure below :



Write down Bernoulli's equation between points 1 (free surface) and 2, between 2 and 3, and between 3 and 4 (free surface again).

6(d) (6 marks) Calculate the volumetric flow rate at which the water is draining from the top tank into the bottom tank.