SOE3152

UNIVERSITY OF EXETER

SCHOOL OF ENGINEERING AND COMPUTER SCIENCE

DEPARTMENT OF ENGINEERING

Fluid Dynamics A

Time allowed : TWO HOURS

January 2002

The marks for this module are calculated from 70% of the percentage mark for this paper plus 30% of the percentage mark for associated coursework.

Full marks may be obtained from full answers to three questions. Candidates have a free choice of questions.

This is a **closed book** examination. Candidates are permitted to use approved portable calculators. A separate formula and data sheet has been provided.

SECTION A

Question 1 (20 marks)

1(a) (4 marks) Define the following terms for a turbine :

i. Hydraulic losses

ii. Type number

iii. No-shock condition

1(b) (4 marks) The properties of a centrifugal pump of diameter 0.35 m running at 1450 RPM (configuration A) are given as follows :

$Q/\text{m}^3\text{s}^{-1}$	0	0.08	0.15	0.23	0.31	0.39	0.46	0.54	0.62
H/m	73.2	74.3	74.0	71.9	69.5	61.5	46.8	26.5	0
η	0	41	60	74	83	83	74	51	0

The manufacturer also supplies a geometrically similar pump of diameter 0.5 m. Predict the performance of this second pump running at 750 RPM (configuration B). Plot both sets of characteristics on the same graph.

1(c) (6 marks) A pipe of diameter D, length L is used to pump water through a vertical height Δz. Show that the characteristic curve for the pipe can be expressed in the form

$$H_L = \Delta z + \frac{KQ^2}{2g} \qquad \text{with} \quad K = \frac{64Lf}{\pi^2 D^5}$$

where Q is the volumetric flow rate, and f the Darcy friction factor.

1(d) (6 marks) Pump A is being used to pump water to a reservoir 28 m above datum, through a pipe of diameter $D = 0.2$ m, length 35 m, with a friction factor $f = 0.004$.

i. What is the volumetric flow rate?

ii. If pump B is used instead, what is the new flow rate?

iii. In each case what is the power being drawn by the pump?

Question 2 (20 marks)

2(a) (10 marks) Explain how an airfoil generates lift.

Data for airfoil NACA3152 is given in the following table :

$\alpha(°)$	-4	0	2	4	6	8	9	10	11	12	12.5
C_L	0.2	0.6	0.81	1.0	1.15	1.3	1.35	1.4	1.46	1.5	1.5
C_D	0.04	0.01	0.0104	0.011		0.013		0.016		0.021	
$\alpha(°)$	13	13.5	14	15	16	17	18	20			
C_L	1.48	1.35	1.3	1.23	1.2	1.17	1.15	1.12			
C_D			0.03	0.036	0.045			0.01			

Plot this data, indicating the salient points of the lift and drag curves.

Describe how this data could have been derived.

2(b) (10 marks) A hydrofoil is a boat with a wing underneath. As the craft picks up speed, the wing generates lift raising the craft out of the water, thus reducing drag. A new "water scooter" is being designed as a single-seat craft for recreational use. It consists of a flat-bottomed craft of length 2 m and width 90 cm, with an airfoil of chord 65 cm and span 90 cm suspended 40 cm below the craft bottom on a fin also of 65 cm length. The total mass of the craft and rider is 250 kg.

i. If the airfoil is set at an angle $\alpha = 5°$, what is the minimum speed necessary for the craft to lift clear of the water?

ii. At this speed, what is the drag on the craft?

iii. By how much has the drag decreased as a result of the craft being clear of the water?

Question 3 (**20 marks**)

3(a) (4 marks) The volumetric flow rate Q of an ideal fluid through a hole of diameter d is believed to depend on the pressure difference Δp and the density of the fluid ρ. Use dimensional analysis to find the relationship between these factors.

3(b) (8 marks) Printing ink (a non-Newtonian liquid) flowing through the cylindrical nozzle of an ink jet printer, radius R, has been found to have the following velocity profile :

$$u = K\left[1 - \left(\frac{r}{R}\right)^3\right]$$

Determine relations for the volumetric flow rate Q and the momentum flux $\mathcal{F}$ through the nozzle, in terms of K. If the mean flow velocity is $\overline{U}$, find an expression for K.

3(c) (4 marks) Ink of density 1100 $\mathrm{kg/m^3}$ flows through a nozzle of diameter 1.2 mm at 20 cm/s. What force is exerted on the nozzle?

SECTION B

Question 4 (**20 marks**)

4(a) (5 marks) Sketch the energy spectrum for homogeneous isotropic turbulence.

4(b) (8 marks) Discuss the important consequences of turbulence in engineering. Your answer should include at least two illustrative examples.

4(c) (7 marks) It has been suggested that the eddy viscosity μ_T near the wall of a pipe can be approximated by the expression

$$\mu_T = \frac{\rho k \overline{u}^2}{\frac{d\overline{u}}{dy}}$$

where k is a constant. The total shear stress is given by

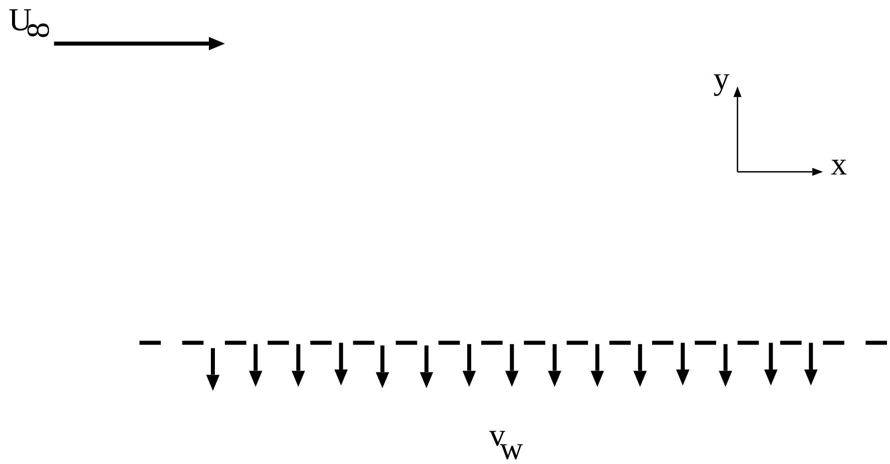
$$\tau = (\mu + \mu_T)\frac{d\overline{u}}{dy}$$

If the total shear stress can be considered constant in the region close to the wall, i.e. $\tau = \tau_0$, find $\overline{u}$ as a function of y, the distance from the wall.

[You may assume that $\displaystyle\int \frac{dy}{a^2 - b^2y^2} = \frac{1}{ab}\tanh^{-1}\frac{by}{a}$]

Question 5 **(20 marks)**

An incompressible fluid is flowing past a flat plate, forming a boundary layer. However the plate is slightly porous, and fluid is being sucked away at a constant speed v_w, as shown in the diagram :



The fluid removal rate is adjusted so that the boundary layer is of a constant thickness, and so the flow is steady and uniform in the x direction.

5(a) (4 marks) What is the value of u_x as $y \to \infty$ (i.e. a long way away from the plate)? Use the continuity equation to show that the y component of the velocity, u_y is uniform throughout the boundary layer and find its value.

5(b) (8 marks) For this case the Navier-Stokes equation for u_x reduces to

$$u_y \frac{\partial u_x}{\partial y} = \nu \frac{\partial^2 u_x}{\partial y^2}$$

Justify this expression. Solve this equation to find an equation for $u_x(y)$.

5(c) (4 marks) The displacement thickness for a boundary layer is given by the expression

$$\delta^* = \int_0^\infty \left(1 - \frac{u_x}{U_\infty}\right) dy$$

Evaluate δ^* for this case.

5(d) (4 marks) What is the drag force on the plate? (Hint : what is the shear stress at the surface?)

Question 6 (20 marks)

A 4 blade Kaplan axial-flow turbine has a hub diameter $D_h = 1.5$ m and a blade diameter $D_t = 3.5$ m. Water flows from the reservoir through a set of inlet guide vanes (wicket gates) which give the flow a tangential component of flow. For the system shown below (see figure on next page), $D_g = 7.0$ m and $w_g = 1.0$ m, whilst $\alpha_g = 30°$. The water is then redirected to flow through the axial turbine. The head driving the flow is the depth of the reservoir, $h = 12$ m, and the turbine discharges to the atmosphere. The turbine turns at 200 RPM at a flow rate $Q = 50$ m^3/s.

6(a) (4 marks) If the exit angle from the wicket gates is α_g, show that the gates impart a whirl velocity

$$v_{w_g} = \frac{Q}{\pi D_g w_g \tan \alpha_g}$$

Find an expression for the mean axial velocity v_f through the turbine in terms of D_h and D_t, assuming the velocity to be independent of r.

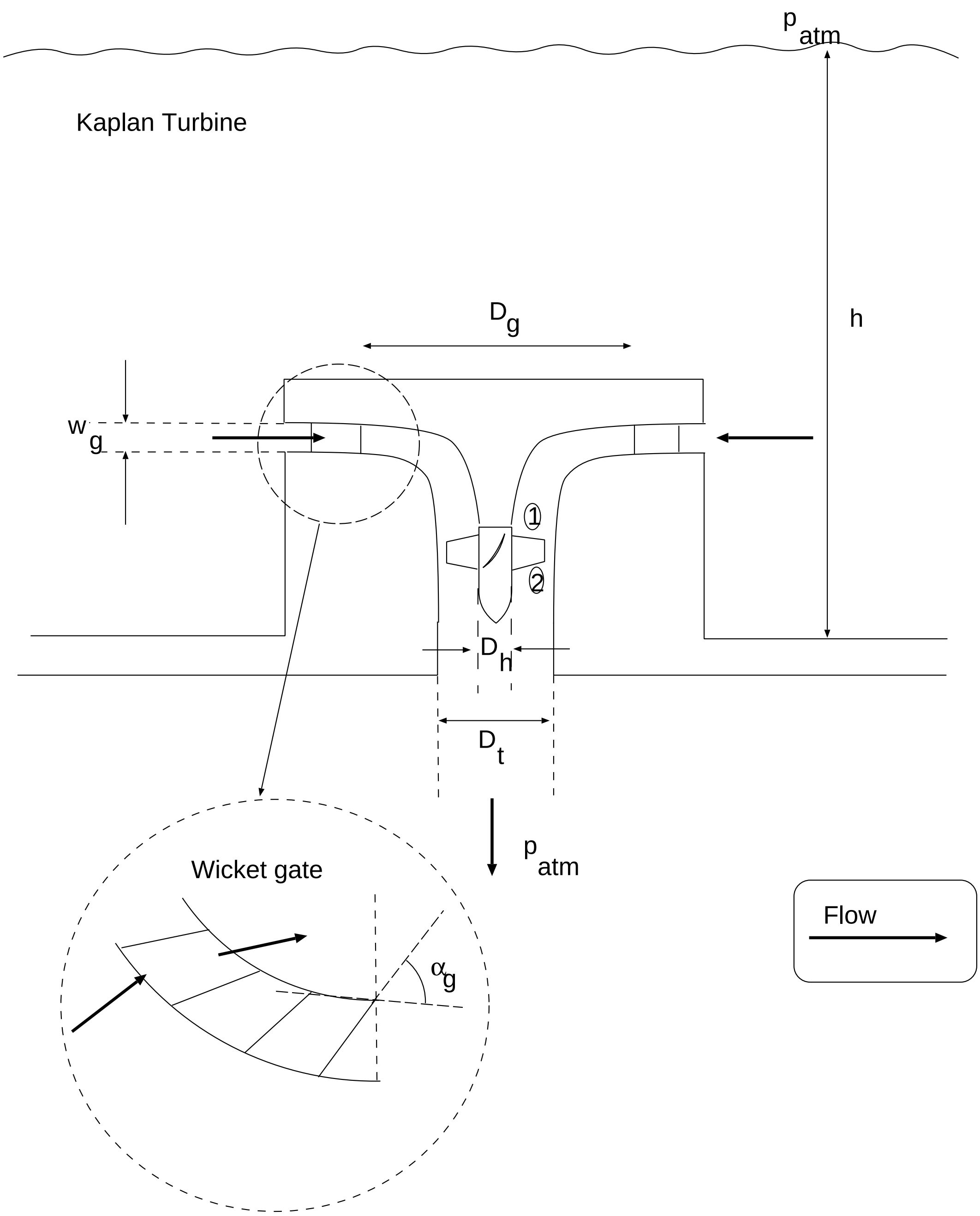
6(b) (6 marks) If the flow between the wicket gates and the turbine has a free-vortex distribution, then the whirl at the turbine is

$$r v_{w_1} = \frac{D_g}{2} v_{w_g}$$

Draw the inlet triangle at $r = 1.2$ m. At what angle β_1 should the turbine blades be set to satisfy the no-shock condition?

6(c) (6 marks) Write Bernoulli's equation between the surface of the reservoir and the impeller inlet (1), and across the impeller itself (hint : across the impeller, streamlines will be rotating with the impeller, and so Bernoulli's equation will involve the relative velocity v_r). Use these to determine the relative velocity with which the water leaves the blade, and the angle at which the blade must be set for the no-shock condition to hold at the outlet.

6(d) (4 marks) Calculate the theoretical power generated by the turbine, if the head decrease across the blades at $r = 1.2$ m is typical of the entire blade. How could this calculation be made more accurate?


p atm
Kaplan Turbine
h
D g
w g
1
2
D h
D t
p atm
Wicket gate
α g
Flow