SOE3047

UNIVERSITY OF EXETER

SCHOOL OF ENGINEERING AND COMPUTER SCIENCE

DEPARTMENT OF ENGINEERING

Advanced Fluid Mechanics

Time allowed: THREE HOURS

September 2001

This is a **closed book** examination.

For **DEFERRED** candidates, the marks for this module are calculated from the better of : 70% of the percentage mark for this paper plus 30% of the percentage mark for associated coursework, or 100% of the percentage mark for this paper.

For **REFERRED** candidates, the marks for this module are calculated from 100% of the mark for this paper. The maximum mark that can be achieved is 40%.

Full marks may be obtained from full answers to **four** questions.

Provided : Formula sheet.

Provided on request : graph paper.

Candidates are permitted to use approved portable calculators.

The allocation of marks to each part of a question is shown thus : **[n]**

Section A.

Question A1. a. The Mk 3 Aerosonde is a small robotic aircraft designed for mid-altitude atmospheric observations. The quoted characteristics of the craft are as follows :

Mass	15 kg	Speed :	Takeoff	18 m/s
Payload	5 kg + 2 kg fuel		Cruise	$20 - 32$ m/s
Altitude	< 6000 m		Climb	4 m/s
Range	4000 km			
Wingspan	2.9 m			
Mean chord	15 cm			

Calculate the lift coefficients for the following regimes :

i. Level flight at 32 m/s at ground level.

ii. Takeoff

iii. Level flight at 5000 m altitude, assuming a speed of 24 m/s. **[8]**

b. How would the design of the wing for the Aerosonde affect its performance? If the lift to drag ratio for the craft aerofoil is 80, how much energy does it expend for a full-range mission? (Assume it spends all its time in regime iii above). **[8]**

c. High altitude atmospheric measurements are often made using helium balloons, which provide a lift force per unit volume $(\rho_{air} - \rho_{He})g$. It is proposed to use a spherical balloon 4 m in diameter to lift a 20 kg instrument package. On release, the balloon quickly settles to a constant rise speed. Calculate this speed. (Take $\rho_{He} = 0.1626$ kg/m^3). **[9]**

Question A2. a. Define the Reynolds number for a cylinder of diameter D whose axis is at right angles to air flowing with velocity V. If the diameter of the cylinder is 4cm what are the Reynolds numbers for $V_1 = 0.035$ mm/s and for $V_2 = 25$ m/s? Sketch the types of flow pattern that you would expect to see as the velocity is increased from V_1 to V_2. **[8]**

b. At a Reynolds number of around 5×10^5 the drag coefficient of a cylinder drops dramatically (see graph 4 in the Formula Sheet). Explain this behaviour. **[4]**

c. A sphere of diameter d moving at constant speed v in oil of density ρ and viscosity μ experiences a viscous resistance force R. Derive dimensionless groups corresponding to R and v, and use these to derive a relationship between R and v. **[7]**

d. Describe 2 ways in which the drag on a body can be reduced, and explain the physical principles involved. **[6]**

Question A3. a. What is meant by the *type number* for a pump? **[3]**

The characteristics of two pumps A (running at 450RPM) and B (running at 550RPM) are given below :

	Pump A		Pump B	
$Q(m^3/s)$	$H(m)$	$\eta(\%)$	$H(m)$	$\eta(\%)$
0	22.6	0	16.2	0
0.006	21.9	32	13.6	14
0.012	20.3	74	11.9	34
0.018	17.7	86	11.6	60
0.024	14.2	85	10.7	80
0.030	9.7	66	9.0	80
0.036	3.9	28	6.4	60

where H is the head developed and η the efficiency of the pump. Plot the characteristics of both pumps, and calculate the type number for each. **[5]**

b. A pipe of diameter D, length L is used to pump water through a vertical height Δz. Show that the characteristic curve for the pipe can be expressed in the form

$$H_L = \Delta z + \frac{KQ^2}{2g} \qquad \text{with} \quad K = \frac{16Lf}{\pi^2 D^5}$$

where Q is the volumetric flow rate, and f the Darcy friction factor. **[8]**

c. One of the pumps in section a. is to be used to pump water continuously through 3.2m of vertical lift, using a pipe 21m long, 10cm in diameter, with a friction factor of 0.02. Select the most suitable pump for this purpose, and justify your choice. **[4]**

What flow rate will your selected pump provide, and what will be the required power input? **[5]**

Section B.

Question B1. a. Sketch the energy spectrum for a turbulent fluid.

Define the time average $\overline{a}$ of a fluctuating quantity a. If a and b are fluctuating quantities, show that

i. $\overline{a+b} = \overline{a} + \overline{b}$

ii. $\overline{ab} = \overline{a}\,\overline{b} + \overline{a'b'}$

where $'$ denotes the fluctuations around the time average. **[9]**

b. A square plate of side 1.5m is dragged through water at 15cm/s. Calculate the drag force on the plate. (Assume that transition occurs at $\mathcal{R}e \sim 10^5$). **[8]**

c. Prandtl's law states that the turbulent shear stress in a fluid is given by the expression

$$\tau = \rho l^2 \left| \frac{\partial \overline{u}}{\partial y} \right| \frac{\partial \overline{u}}{\partial y}$$

where near a wall the Prandtl length scale $l = \kappa y$, y being the distance from the wall. Use this to derive an expression for the velocity profile close to the wall (the *log law* profile). **[8]**

Question B2. A centrifugal turbine with inside diameter 40cm and outside diameter 1 m rotates at 200RPM and discharges 0.8 m^3/s of water. The blade height is 10 cm, and the water inlet imparts a swirl velocity v_{w1} of 1.45 m/s.

a. Draw a vector triangle for the inlet velocities. Calculate the relative and absolute inlet velocities. What angle should the blades be set at for the no-shock condition to hold? **[7]**

b. If the blades sweep around to an angle $\beta_2 = 40°$, what are the flow velocities (absolute and relative) at the outlet? Draw the vector triangle at the outlet. **[8]**

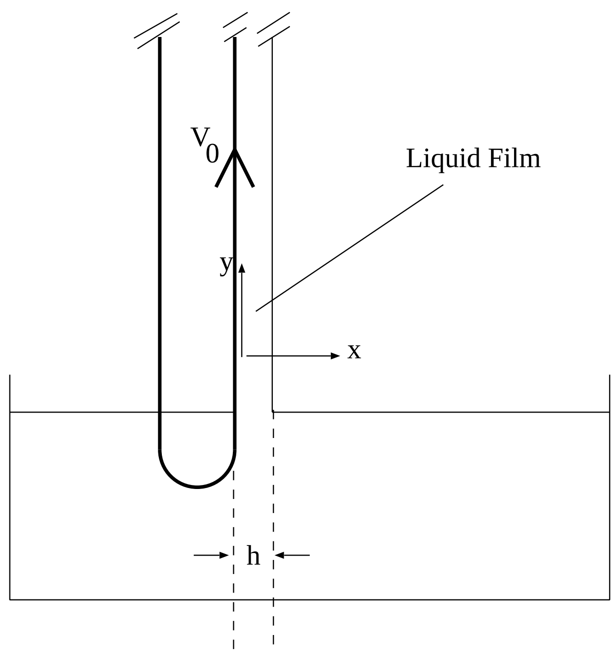
c. The turbine runs using water supplied from a reservoir 14 m high, through a 30 m long pipe 40 cm in diameter, inside roughness of 0.001. If the lost head in the connections is 0.45 m of water, calculate the hydraulic efficiency for the turbine. What factors might account for the drop in efficiency? **[10]**

Question B3. a. Show that the velocity

$$\underline{u} = 2x^2 z\underline{i} - 4xyz\underline{j} + 3x\underline{k}$$

could represent flow of an incompressible fluid. **[3]**

b. A broad belt passes vertically upwards through a tank of liquid at constant velocity V_0 and picks up a liquid film. The effect of gravity is to make the liquid drain back into the tank, but, at a sufficiently high belt velocity it is observed that, at a short distance above the liquid surface the film thickness becomes constant and equal to h. In this region of constant film thickness it may be assumed that the flow is fully developed.



Assuming steady flow and negligable shear at the liquid/air interface, show that the Navier-Stokes equations reduce to

$$0 = -g + \nu \frac{d^2 u_y}{dx^2}$$

with ν the kinematic viscosity and u_y the y component of velocity (take the y axis to lie along the belt). **[6]**

c. Write down the boundary conditions at the belt and at the liquid/air interface. **[4]**

d. Show that the velocity profile in the developed region is given by

$$\frac{u_y}{V_0} = 1 - \frac{gh^2}{\nu V_0}\left[\frac{x}{h} - \frac{1}{2}\left(\frac{x}{h}\right)^2\right]$$

[6]

e. Determine the net volumetric flow rate per unit breadth of the belt. **[6]**