Question 1. Solution.

b.

This gives characteristic data as follows :

$Q/\text{m}^3\text{s}^{-1}$	0	0.12	0.23	0.35	0.47	0.58	0.70	0.81	0.93
H/m	40.0	40.6	40.4	39.3	38.0	33.6	25.6	14.5	0
η	0	41	60	74	83	83	74	51	0

d. $Q_A = 0.42\ \text{m}^3/\text{s}$, and $Q_B = 0.28\ \text{m}^3/\text{s}$. The efficiency is about the same in both cases, at around $\eta = 80\%$. $H_A = 55\text{m}$, so $P_A = 280$ W. $H_B = 40\text{m}$, so $P_B = 140$ W.

Question 2. Solution

a. The data should give a graph showing a linear variation of lift with angle (small angle approximation), and a non-linear section, followed by stall (drop in lift, sharp increase in drag).

b. i. Lift velocity 2.76 m/s.

ii. The drag will be comprised of two elements : drag on the airfoil and drag on the fin. For the airfoil, $F_D = 27$ N. The drag on the fin is due to the boundary layer. Assuming that the b.l. is turbulent from the leading edge, the force on the b.l. is $F_{bl} = 4.11$ N. Of course there will be 2 sides to this, so the total drag will be 35 N.

iii. If we include the skin friction drag on the underside of the craft, the force is $F_u = 36$ N. In other words, the drag has more than halved as a result of this.

Question 3. Solution.

a.

$$Q = Kd^2\sqrt{\frac{\Delta p}{\rho}}$$

b. For the volumetric flux

$$Q = \int u dA = \frac{3\pi K R^2}{5}$$

For the momentum flux

$$\mathcal{F} = \int \rho u^2 dA = \frac{17\pi R^2 \rho K}{35}$$

The mean velocity is $\overline{U} = Q/A$, so from the first expression

$$\overline{U} = \frac{3\pi K R^2/5}{\pi R^2} = \frac{3}{5}K \quad \Rightarrow \quad K = \frac{5}{3}\overline{U}$$

c. Substituting for K in the momentum expression

$$\mathcal{F} = \frac{17}{21}\pi R^2 \rho \overline{U} = 2.0 \times 10^{-4}\mathrm{N}$$

This is the rate of flow of momentum from the nozzle, so must equal the force exerted by the jet of ink on the nozzle.

Question 4. Solution.

c. Substituting for μ_T gives

$$\begin{aligned}\tau_0 &= \left(\mu + \frac{\rho k \overline{u}^2}{\frac{d\overline{u}}{dy}}\right)\frac{d\overline{u}}{dy} \\ &= \mu\frac{d\overline{u}}{dy} + \rho k \overline{u}^2\end{aligned}$$

rearanging this gives the equation

$$\mu\frac{d\overline{u}}{dy} = \tau_0 - \rho k \overline{u}^2$$

and so

$$\int \frac{d\overline{u}}{\tau_0 - \rho k \overline{u}^2} = \int \frac{dy}{\mu}$$

The l.h.s. corresponds to the integral given in the question, with $a^2 = \tau_0$ and $b^2 = \rho k$. Thus the solution is

$$\frac{y}{\mu} = \frac{1}{\sqrt{\tau_0 \rho k}} \tanh^{-1}\sqrt{\frac{\rho k}{\tau_0}}\overline{u} + C$$

Of course, where $y = 0$ one would expect $\overline{u} = 0$, which gives $C = 0$, so we can invert this

$$\overline{u} = \sqrt{\frac{\tau_0}{\rho k}} \tanh \frac{y\sqrt{\tau_0 \rho k}}{\mu}$$

Question 5. Solution.

a. As $y \to \infty$, $u_x \to U_\infty$.

b.

$$u_x = U_\infty\left(1 - \exp\left(-\frac{v_w}{\nu}y\right)\right)$$

c.

$$\delta^* = \frac{\nu}{v_w}$$

d. The shear stress is

$$\begin{aligned}\tau &= \rho\nu\frac{\partial u_x}{\partial y}\\ &= \rho\nu U_\infty \frac{v_w}{\nu}\exp\left(-\frac{v_w}{\nu}y\right)\end{aligned}$$

Evaluating this at $y = 0$ gives

$$\tau_0 = \rho U_\infty v_w$$

The drag force is therefore this $\times$ LD, where L is the length of the plate and D its breadth.

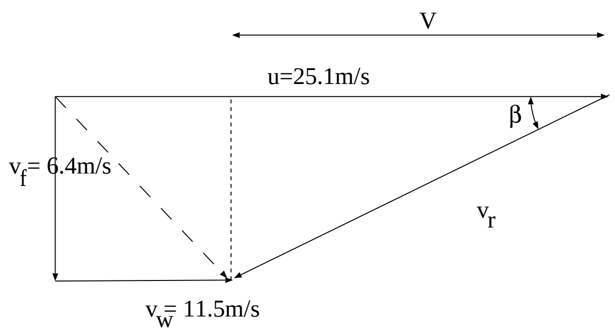
$$F = \rho U_\infty v_w LD$$

Question 6. Solution.

a.

$$v_f = \frac{Q}{\pi/4\,(D_t^2 - D_h^2)}$$

b. The inlet triangle looks something like this :



$\beta = 25.2°$.

c. Bernoulli from the surface to the impeller is

$$p_{atm} + \rho g h = p_1 + \frac{1}{2}\rho v_1^2$$

Across the blade

$$p_1 + \frac{1}{2}\rho v_{r_1}^2 = p_{atm} + \frac{1}{2}\rho v_{r_2}^2$$

Eliminating p_1 between these,

$$v_{r_2}^2 = 2gh + v_{r_1}^2 - v_1^2 = 2\times 9.81\times 12 + 15.06^2 - 13.14^2 = 289.6$$

and $v_{r_2} = 17$ m/s. $\beta_2 = 34.4°$.

d. Eulers equation for the theoretical head

$$H = \frac{1}{g}\left(u_2 v_{w_2} - u_1 v_{w_1}\right)$$

$$P = \rho g Q H = 1000 \times 9.81 \times 50 \times 10.95 = 5.4 \text{ MW}$$

For full accuracy, one would take into account the variation of the flow properties across the blade. Also power losses, friction etc would be important in a real turbine.