Question A1. Solution.

a.

i. Level flight at $U = 32\text{m/s}$

$$C_L = 0.79$$

ii. Takeoff. We will assume that the lift still balances the weight of the plane, although the plane will be accellerating upwards

$$C_L = 2.5$$

iii. At 5000m altitude, the density is lower

$$C_L = 2.34$$

b. Since $C_L/C_D = 80$, the drag coefficient for cruising at altitude is therefore 0.02925. If the range is 4000km, then the energy expended is 1×10^7J.

c. The forces on the balloon are 1. gravity (W), 2. upthrust (F), and 3. air drag. This last varies with the rise velocity, which is why the balloon will reach a steady rise velocity. Iterate to find the rise speed to be 11m/s. Exact results will depend on the care taken reading the graph.

Question A2. Solution

b.

The head lost in a pipe is given by the Darcy-Weisbach formula. From this write an expression for the head lost in a section dL

$$dH_L = \frac{8Q^2 f}{g\pi^2} \frac{dL}{D^5}$$

The diameter D varies with L, so integrate

$$H_L = \int_0^4 dH_L = \frac{8Q^2 f}{g\pi^2} \int_0^4 \frac{dL}{(0.4 - 0.05L)^5} = 0.121\text{m}$$

Question A3. Solution.

a. Sketches showing creeping flow, recirculating flow, von Karman vortex shedding, turbulent wake.

b. At this $\mathcal{R}e$, $C_D \sim 1.1$, so the force per bar is

$$F_D = 8.20\text{N}$$

c. Thus the total force on the roofrack (ignoring the frame) will be 41N. The roof itself is $2\text{m} \times 1.8\text{m} = 3.6\text{m}^2$, and the boundary layer is probably turbulent. Here $\mathcal{R}e_L = 3 \times 10^6$, so

$$\overline{C_f} = \frac{0.074}{\mathcal{R}e_L^{1/5}} = 0.00376$$

so

$$F_r = 4\text{N}$$

and so the ratio of the drags is 10.

d. $f = 313\text{Hz}$

e. Laminar boundary layer = region in which the flow velocity goes to the wall velocity (generally 0). Blasius solution. Turbulent boundary layer = 3 sections, laminar sublayer, transition layer, log-law region. Law of the Wall.

Question B1. Solution.

a. The curve is actually a straight line giving a lift coefficient of 0.6 for $\alpha = 0$ and 1.3 for $\alpha = 5°$. This is not uncommon (small angle approximation). Beyond $\alpha = 5°$ the curve may stop being linear, and will definitely reach a maximum and drop off. The maximum is the stall angle, and is caused by vortex shedding from the leading edge of the airfoil.

b. The force exerted on an airfoil is

$$F_L = \frac{1}{2}\rho U^2 A C_L$$

with A the plan area of the airfoil. However the different pieces of the rotor will be moving at differing speed. So we will consider a section of the blade $dA = cdr$:

$$dF = \frac{1}{2}\rho\Omega^2 c C_L r^2 dr$$

For 4 blades the total force is therefore

$$F_T = \int_0^R 4dF = \frac{1}{2}\left(4\rho\Omega^2 c R^3\right)\int_0^1 C_L y^2 dy$$

This is in the desired form, with

$$q = \left(4\rho\Omega^2 c R^3\right)$$

c. The angle of attack is

$$\alpha(r) = \alpha_0\left(1 - \frac{r}{2R}\right), \qquad \alpha_0 = 5°$$

The lift varies as

$$C_L(\alpha) = 0.6\,(1 + 0.233\alpha) \qquad (\alpha \text{ in degrees})$$

Substituting for α we find

$$C_L(\alpha) = 0.6(2.165 - 0.5825y)$$

Doing the integral :

$$\int_0^1 C_L y^2 dy \quad = \quad 0.3456$$

The force on the helicopter is 7651.8N. Rearanging, this gives

$$q = 2\frac{F_T}{0.3456} = 44281.25$$

Thus

$$\Omega^2 = \frac{q}{4\rho c R^3} = \frac{44281.25}{4 \times 1.2 \times 0.12 \times 6^3} = 355.9$$

This gives $\Omega = 18.87$/s, or 180RPM.

Question B2. Solution.

a. For an axial flow turbine, the inlet and outlet flows are both along the axis of rotation of the turbine. For a centrifugal flow turbine, the flow is turned through $90°$ and exits in the plane normal to this axis.

i. Overall efficiency for a pump is the ratio of fluid power output from the machine to the power input to the shaft.

ii. Hydraulic losses are the fluid flow losses (as against frictional losses in bearings etc) – basically the sum of impeller loss (shocks) leakage loss (fluid missing the impeller) and casing loss (fluid drag against the casing).

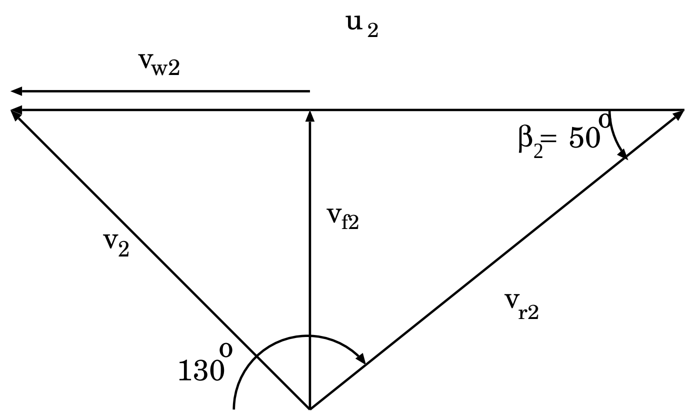
iii. No shock condition implies the fluid meets or leaves the impeller blades tangentially, ie $\beta' = \beta$.

For a pump, the dimensionless quantities of importance are K_Q and K_H. The type number is constructed to eliminate the diameter D of the impeller, ie to remove scale effects.

$$n_s = \frac{K_Q^{1/2}}{K_H^{3/4}} = \left(\frac{Q}{ND^3}\right)^{1/2} / \left(\frac{gH}{N^2D^2}\right)^{3/4} = N\frac{Q^{1/2}}{(gH)^{3/4}}$$

thus independent of size.

b.i. Outlet triangle :



If no ke. removed

$$(\text{Work done by impeller}) = (\text{Static head}) + (\text{Velocity head})$$

ie.

$$\frac{u_2 v_{w2}}{g} = H + \frac{v_2^2}{2g} \tag{1}$$

Thus

$$u_2 = 19.98\text{m/s}$$

Since $u_2 = r^2\omega$ and $\omega = 83.78\text{rad/s}$, $r_2 = u_2/\omega = 0.238\text{m}$, and the diameter is therefore 48cm.

ii. If 40% of the ke. is recovered, then 60% is wasted. This implies (1) becomes

$$\frac{u_2 v_{w2}}{g} = H + 0.6\frac{v_2^2}{2g}$$

giving a quadratic for u_2

$$1.4u_2^2 - 1.3425u_2 - 396.49 = 0$$

Solving this quadratic

$$u_2 = \frac{+1.3425 \pm \sqrt{(1.3425)^2 + 4 \times 1.4 \times 396.49}}{2 \times 1.4} = 17.32\text{m/s}$$

taking the obvious root. Thus $r_2 = 0.207\text{m}$ and the diameter is now 41cm.

Question B3. Solution.

a. There are 6 variables in the problem, and 3 dimensions (M, L, T). Hence there are 3 dimensionless groups. Taking D, ρ, μ as repeating variables will give groups for Q, Δp and D_0, which seems reasonable. This implies an expression linking the variables of the form

$$\frac{Q\rho}{D\mu} = f\left(\frac{\mu^2 \Delta p}{D^6 \rho^3}, \frac{D_0}{D}\right)$$

b. Since the flow is fully developed,

$$\frac{D}{Dt} = 0$$

In addition, the flow is driven by gravity, so $\nabla p = 0$. Thus the NS equations reduce to

$$0 = \nu \nabla^2 \underline{u} + \underline{F}$$

c. We can assume that the flow is parallel to x, so $u_x(y)$ is the only possible component. Resolving the gravitational force along x gives $F_x = \rho g \sin\alpha$, so the equation becomes

$$\frac{d^2 u_x}{dy^2} = -\frac{g \sin\alpha}{\nu}$$

Boundary conditions are that on the plane $y = 0$, $u_x = 0$, whilst on the plane $y = h$, $\frac{du_x}{dy} = 0$. Integrating the expression gives

$$\frac{du_x}{dy} = -\frac{g \sin\alpha}{\nu} y + A$$

for which the 2nd b.c. gives

$$A = \frac{g \sin\alpha h}{\nu}$$

so

$$\frac{du_x}{dy} = \frac{g \sin\alpha}{\nu}(h - y)$$

Integrating again

$$u_x = \frac{g \sin\alpha}{\nu} \int (h - y) dy + B = \frac{g \sin\alpha}{\nu}\left(hy - \frac{y^2}{2}\right) + B$$

Since $u_x = 0$ at $y = 0$, $B = 0$, giving the profile

$$u_x = \frac{g \sin\alpha}{\nu}\left(hy - \frac{y^2}{2}\right)$$

as required.

d. Integrating this profile

$$Q = \int_0^h u_x dy \quad = \quad \frac{gh^3 \sin\alpha}{3\nu}$$