### Law of Indices (Ref: Croft & Davison Ch.7)

Terms that have the same base but different powers are different terms and they cannot be added or subtracted.

E.g. \(2a^2\) cannot be combined with \(3a^3\) by adding or subtracting but provided they have the same BASE they can be multiplied or divided.

E.g. \(a^2 \times a^3 = a^5\)

Remember the power or index only tells you how many times a number or letter is to be multiplied by itself.

So \(a^2\) really means \(a \times a\) and \(a^3\) really means \(a \times a \times a\),

\(a^2 \times a^3\) really means \(a \times a \times a \times a \times a\) so \(a^2 \times a^3 = a \times a \times a \times a \times a = a^5\)

But the power of 5 in the answer to \(a^2 \times a^3\) can be found by adding the powers of the two terms. This is called the LAW of INDICES.

### Multiplication

Law of Indices states that provided the bases are the same, algebraic terms can be multiplied by adding the Indices.

E.g. \(x^2 \times x^4 = x^{(2+4)} = x^6\)

If the terms to be multiplied have coefficients, they are multiplied as normal

E.g. \(3x^2 \times 4x^3 = 12x^{(2+3)} = 12x^5\)

Remember the rules covering directed numbers

E.g. \(5x^2 \times -2x^{-3} = 10x^{(5+(-3))} = 10x^2\)

If the bases are not the same, only the coefficients can be combined

E.g. \(3a^2 \times 2b^3 = 6a^2b^3\)

### Division

Law of Indices states that provided the bases are the same, algebraic terms can be divided by subtracting the indices.

E.g. \(x^5 \div x^2 = x^{(5-2)} = x^3\)
Coefficients of terms are divided as normal

eg. \(24x^4 \div 6x^2 = 4x^{(4-2)} = 4x^2\)

Care must be taken if the terms have NEGATIVE indices

eg. \(x^4 \div x^{-3} = x^{(4-(-3))} = x^7\)

**Raising to a power**

Sometimes terms that already include a power or index must be raised to a further power, in this case the two powers are multiplied

eg. \((a^2)^3\) This means \(a^2\) must be multiplied by itself 3 times \(a^2 \times a^2 \times a^2 = a^{(2 \times 3)} = a^6\)

If the term has a coefficient it must be raised to the power

eg. \((3x^3)^4 = 81x^8\) or \((2x^{-3})^5 = 32x^{-15}\)

**Lowering to a root**

The term \(36x^2\) is an example of an exact square - this means that it is something multiplied by itself, in this case \(6a\) because \(6a \times 6a = 36a^2\). So \(6a\) is said to be the square root of \(36a^2\)

The cube root of \(27a^3\) would be \(3a\) because \(3a \times 3a \times 3a = 27a^3\). Finding the root of a number is called lowering the number to a specified root.

To lower a term to a root the power of the term is divided by the root required

eg. \(2\sqrt{x^6} = x^{(6 \div 2)} = x^3\)

If the term has a coefficient the root must be found for this as well as for the symbol.

eg. \(3\sqrt{125x^{12}} = 5x^{(12 \div 3)} = 5x^4\) or \(5\sqrt{32b^{20}} = 2b^4\)

**Fractional and Negative Powers**

Fractional powers are another way of expressing roots eg \(x^{\frac{1}{4}}\) is another way of saying the fourth root of \(x\) or \(a^{\frac{1}{15}}\) is the same as the fifth root of \(a\).

If an algebraic term has a negative index, it can be converted into a positive index by taking the reciprocal of the term and changing the sign if the index
e.g. \( x^{-2} = \frac{1}{x^2} \) or \( \frac{5}{x^3} = 5x^{-3} \)

Care must be taken in identifying the elements of a term that any power applies to.

e.g. in \( 5x^3 \) the index applies to the \( x \) element only

in \((5x)^3\) the index applies to the 5 and the \( x \) elements

This is important when taking reciprocals and changing the sign of the index.

e.g. \( 4x^{-2} = \frac{4}{x^2} \) the negative power only applies to the \( x \) element so only this moves to

the denominator when the sign of the negative index is changed to a positive value.

### Law of indices - Worksheet 1

Simplify the following:

1. \( x^2 \times x^4 = \)
2. \( a^3 \times x^5 = \)
3. \( p^3 \times p^2 = \)
4. \( y^3 \times y^4 = \)
5. \( a^5 \times a^4 \times a^3 = \)
6. \( p^3 \times p^2 \times p^4 = \)
7. \( x^2 \times x^3 \times x^4 \times x^{-2} = \)
8. \( a^2 \times a^4 \times a^2 \times a^5 = \)

### Law of indices - Worksheet 2

Simplify the following:

1. \( 9x^5 \div 3x^2 = \)
2. \( 8x^3 \div 2x^5 = \)
3. \( y^3 \times y^2 + y^3 = \)
4. \( x^4 \times x^3 + x^5 = \)
5. \( (x^3)^2 = \)
6. \( (y^5)^3 = \)
7. \( (2a^3)^4 = \)
8. \( (3y^{-2})^2 = \)