SOE3047

UNIVERSITY OF EXETER

SCHOOL OF ENGINEERING AND COMPUTER SCIENCE

DEPARTMENT OF ENGINEERING

Advanced Fluid Mechanics

Time allowed: THREE HOURS

August/September 2000

This is a **closed book** examination.

For **DEFERRED** candidates, the marks for this module are calculated from the better of : 80% of the percentage mark for this paper plus 20% of the percentage mark for associated coursework, or 100% of the percentage mark for this paper.

For **REFERRED** candidates, the marks for this module are calculated from 100% of the mark for this paper. The maximum mark that can be achieved is 40%.

Full marks may be obtained from full answers to **four** questions.

Provided : Formula sheet.

Provided on request : graph paper.

Candidates are permitted to use approved portable calculators.

The allocation of marks to each part of a question is shown thus : **[n]**

Question 1. What does it mean to say that the flow in a pipe is 'fully developed'? **[3]**

Air at 30° C and atmospheric pressure enters a pipe of 2cm diameter and 2m length with a uniform velocity $U_0 = 1.2\text{m/s}$. If the entrance length is given by

$$L_e/D = 0.0288\mathcal{R}e$$

calculate L_e. **[2]**

Draw a sketch of the pipe, marking the edge of the boundary layer and indicating the velocity profiles at the entrance and at L_e. **[3]**

If the velocity distribution at L_e is described by Poiseuille's law, derive an expression for the volumetric flow rate Q. **[4]**

Calculate the centreline velocity at L_e. **[4]**

What is the pressure drop between the two ends of the pipe? **[4]**

Question 2. a. Sketch and annotate the energy spectrum of a turbulent fluid. **[4]**

b. Define the following quantities :

i. The Kolmogorov length scale

ii. Turbulent Kinetic Energy **[4]**

c. Define the time average $\overline{a}$ of a fluctuating quantity a. If a and b are fluctuating quantities, show that

i. $\overline{a+b} = \overline{a} + \overline{b}$

ii. $\overline{ab} = \overline{a}\overline{b} + \overline{a'b'}$

where $'$ denotes the fluctuations around the time average. **[5]**

d. Two infinitely long parallel plates at $y = \pm h$ apart move in opposite directions with velocities $u_y = \pm U_0$. The fluid flow between the plates is turbulent. The turbulent shear stress can be described by Prandtl's mixing length formula

$$\tau_t = \rho l^2 \left|\frac{\partial \overline{u}}{\partial y}\right| \frac{\partial \overline{u}}{\partial y}$$

with

$$l(y) = K(h^2 - y^2)$$

If the turbulent shear stress is $\tau_t = \rho u_*$ where u_* is the friction velocity, derive an expression for the velocity profile $\overline{u}/u_*$. You may assume that

$$\int \frac{dy}{h^2 - y^2} = \frac{1}{2h} \ln \left|\frac{h+y}{h-y}\right|$$

[7]

Question 3. a. The volumetric flow Q of fluid through a circular orifice depends on the density of the fluid ρ, the area of the orifice A and the pressure drop across the orifice Δp. Using dimensional analysis, derive an expression for Q. **[6]**

b. The characteristic curve for a centrifugal pump is given by the manufacturer as

$$H = 40 - 140Q - 4200Q^2$$

with the pump operating at a speed of 800 RPM. Obtain an expression for the pump characteristic for the same pump operating at 75% of its normal operating speed. Plot both sets of characteristics for Q values up to 0.05 m^3/s. **[6]**

c. Running at full speed, the pump is used to pump liquid along a pipe of length 100 m and diameter 16 cm, and discharges from the end of the pipe at atmospheric pressure at an elevation of 10 m above the open feed tank. Determine the duty point for the system. (Assume a friction factor of 0.02 for the pipe and neglect entrance and exit losses). **[8]**

Question 4. A centrifugal turbine with inside diameter 40cm and outside diameter 1m rotates at 200RPM and discharges $0.8m^3/s$ of water. The blade height is 10cm, and the water inlet imparts a swirl velocity v_{w1} of 1.45m/s.

a. Draw a vector triangle for the inlet velocities. Calculate the relative and absolute inlet velocities. What angle should the blades be set at for the no-shock condition to hold? **[6]**

b. If the blades sweep around to an angle $\beta_2 = 40°$, what are the flow velocities (absolute and relative) at the outlet? Draw the vector triangle at the outlet. **[6]**

c. The turbine runs using water supplied from a reservoir 14m high, through a 30m long pipe 40cm in diameter, inside roughness of 0.001. If the lost head in the connections is 0.45m of water, calculate the hydraulic efficiency for the turbine. What factors might account for the drop in efficiency? **[8]**

Question 5. a. Sketch the different flow patterns you would expect to see behind a cylinder at $\mathcal{R}e$ between 0.1 and 50,000. **[5]**

b. A car roofrack consists of 5 cylindrical bars 1.8m in length and 1.4cm in diameter. The roof is 2m in length. If the car is driven at 80km/hr, what is the drag force exerted on each bar? **[3]**

c. What is the ratio of the the drag on the roofrack to the drag on the roof? **[5]**

d. At what frequency do the bars vibrate? **[2]**

e. How can drag be reduced on a moving body? **[5]**

Question 6. a.How does an airfoil generate lift? Discuss the following aspects of flight :

i. Stall.

ii. Wing flaps. **[7]**

b. Plot the polar diagram for the Lissamann 7769 airfoil (data below). **[2]**

c. The Gossamer Albatross was the first human-powered aircraft to cross the English Channel. It had a wing of constant cross-section based on the Lissaman 7769 airfoil, of total width (wingtip to wingtip) 28 m, chord length 1.8 m, and a thin cockpit 2.5 m high and 3 m long containing the pilot. The total mass of the craft was 97.5 kg. The wings were set at an angle of 10°. How fast did the craft have to move in level flight to remain airborne? **[3]**

d. What fraction of the drag is accounted for by the fusilage if construction gives imperfections of 2.5 mm height? If the fusilage can be made hydraulically smooth, what does this value drop to? **[6]**

e. How much energy did the pilot expend crossing the Channel (distance 36km)? **[2]**

Lissamann 7769 Airfoil data.

α	C_L	C_D	α	C_L	C_D
-5.00	-0.22	0.0391	11.00	1.33	0.0409
-4.00	-0.14	0.0355	12.00	1.40	0.0460
-3.00	-0.05	0.0328	13.00	1.46	0.0509
-2.00	0.05	0.0113	14.00	1.50	0.0573
-1.00	0.16	0.0114	15.00	1.52	0.0646
0.00	0.28	0.0113	16.00	1.52	0.0736
1.00	0.36	0.0140	17.00	1.50	0.0831
2.00	0.46	0.0155	18.00	1.46	0.0973
3.00	0.56	0.0166	19.00	1.38	0.1258
4.00	0.67	0.0180	20.00	1.29	0.1592
5.00	0.77	0.0182	21.00	1.22	0.1834
6.00	0.86	0.0203	22.00	1.15	0.2049
7.00	0.96	0.0226	23.00	1.10	0.2159
8.00	1.05	0.0253	24.00	1.03	0.2388
9.00	1.14	0.0283	25.00	0.96	0.2633
10.00	1.25	0.0310			