Weirs and Flumes

SOE2156: Fluids Lecture 7

7.1 Recap

Last lecture – assumed the channel was uniform (constant depth, shape, slope etc.) – *steady uniform flow*

Found that :

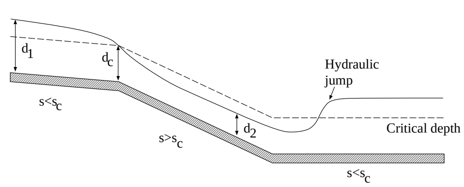
- location of free surface to be determined
- 2 possible solutions – subcritical and supercritical. Solution can jump between these (hydraulic jump)
- Froude number important
- flow can also be *critical* – $\mathcal{F}r = 1$
- Single solution for this case : maximum flow for a given specific energy

What if the channel is non-uniform? – *steady non-uniform flow*?

7.2 Steady non-uniform flow

Froude number still indicates whether the flow is sub- or super-critical. As the cross-section changes, the flow conditions will too.

The free surface may pass through the critical depth. Points where this happens are a limiting factor in the design of the channel, and are known as *control sections*.

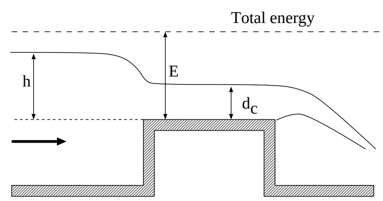


Examples of this behaviour include weirs and flumes, used (principally) to measure flow rates of water.

Weirs are vertical obstructions – can be thin or broad-crested. Flumes are narrowings in the channel.

7.3 Broad crested weirs

Typical arangement :



- Sufficiently broad that the free surface is parallel to the crest
- Flow upstream tranquil – $\mathcal{F}r < 1$
- Flow downstream unimpeded – free jet – discharge over the weir will be the maximum possible
- Thus flow over weir at critical depth

Flow rate over weir given by

$$Q = CbE^{3/2}$$

in terms of the upstream specific energy E. Usually assume $V = 0$ upstream, so $E \simeq H$, and thus

$$Q = CbH^{3/2}$$

– *general weir equation*. A single measurement of this value gives the discharge. From theoretical analysis

$$C = 1.705$$

but this is usually determined from experiment.

N.B. The depth over the crest of the weir is fixed. Raising the weir crest will not alter this, but will alter the overall depth upstream.

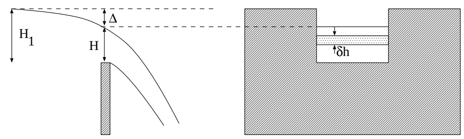
7.4 Thin weirs

Alternative arangement – weir is thin (flat plate) with specified opening.

Assumptions :

- Flow behind weir $V = 0$
- Pressure in nappe atmospheric
- No energy losses
- Streamlines horizontal over crest

7.5 Rectangular notch weir



Start from

$$Q = AV$$

Consider the thin strip marked

$$A = b\delta h$$

Applying Bernoulli gives

$$V = \sqrt{2gh}$$

so

$$\delta Q = b\delta h\sqrt{2gh}$$

and thus

$$Q = \int_0^H dQ = \int_0^H b\sqrt{2gh}\, dh = b\sqrt{2g}\frac{2}{3}H^{3/2}$$

N.B. This is wrong : we usually measure H_1, not H A more detailed analysis can be carried out, with $\Delta \simeq H_1/3$, giving

$$Q = 0.81\left(\frac{2}{3}\right)\sqrt{2g}\; bH_1^{3/2}$$

This is still inaccurate : we have neglected the velocity head upstream (important for small channels). In reality, just use the formula

$$Q = \left(\frac{2}{3}\right) C_d\sqrt{2g}\; bH^{3/2}$$

where we have assumed $H_1 \simeq H$

C_d is a coefficient to be determined for each weir. Generally $C_d \simeq 0.62$

7.6 Vee notch weir

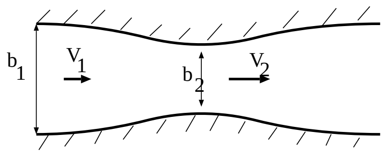
A similar analysis for a weir of angle θ is possible. In this case the width of the strip

$$b = 2(H - h)\tan\theta/2$$

and the simple analysis gives

$$Q = \frac{8}{15}\sqrt{2g}\tan\frac{\theta}{2}H^{5/2}$$

7.7 Flumes



. . . plan view

Width of channel reduced ⇒ flow speeds up ⇒ discharge per unit width increases.

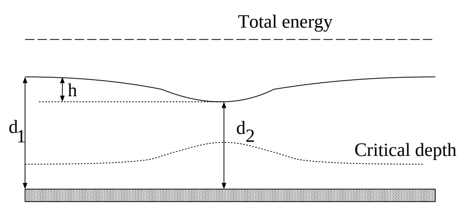
Specific energy constant

⇒ for subcritical flow, depth decreases

⇒ for supercritical flow, depth increases

This arrangement can be used for flow measurement.

If the free surface does not pass through the critical depth, we call this a *venturi flume*



If the free surface passes through the critical depth in the throat, then we have a *standing wave flume*