Basic Mathematics

Solving Simple Equations

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The aim of this document is to provide a short, self assessment programme for students who wish to acquire a basic competence at solving simple equations.

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1. Introduction

In this section we shall look at some simple equations and the methods used to find their solution. There are four basic rules:

Rule 1 An equal quantity may be added to both sides of an equation.

Rule 2 An equal quantity may be subtracted from both sides of an equation.

Rule 3 An equal quantity may multiply both sides of an equation.

Rule 4 An equal, *non-zero* quantity may divide both sides of an equation.

The application of these rules is illustrated in the following examples.

Example 1 Solve the equations

$$\text{(a)} \quad 3x - 8 = x + 10\,, \qquad \text{(b)} \quad \frac{x}{2} = -6\,.$$

Solution

(a) By Rule 1 we may add 8 to both sides:

$$3x - 8 + 8 = x + 10 + 8 \quad \text{i.}e. \quad 3x = x + 18\,.$$

By Rule 2 we may subtract x from both sides:

$$3x - x = x + 18 - x \quad \text{i.}e. \quad 2x = 18\,.$$

Finally, by Rule 4 we may divide both sides by 2 giving $x = 9$.

(b) By Rule 3 we may multiply both sides by 2,

$$\left(\frac{2}{1}\right) \times \left(\frac{x}{2}\right) = 2 \times (-6) \quad \text{i.}e. \quad x = -12\,.$$

It is always good to check that the solution is correct by substituting the value into both sides of the equation. In **Example 1** (a), by substituting $x = 9$ into the left hand side of the equation we see that

$$3x - 8 = 3 \times 9 - 8 = 19\,.$$

Substituting $x = 9$ into the right hand side of the equation gives

$$x + 10 = 9 + 10 = 19\,.$$

Since both sides of the equation are equal when $x = 9$, it is a correct solution. In this case it is the only solution to the equation but it is important to note that some equations have more than one solution.

EXERCISE 1. Solve each of the following equations. (Click on green letters for solutions.)

(a) $3x = 18$,
(b) $7x = -14$
(c) $-2x = -10$
(d) $28x = 35$
(e) $5x - 3x - 12x = 29 - 2 - 7$
(f) $-\dfrac{x}{5} = 3$

Try the following short quizzes.

Quiz Which of the following is the solution to the equation

$$8x + 5x - 3x = 17 - 9 + 22\,?$$

(a) 2 (b) -2 (c) 3 (d) -3

Quiz Which of the following is the solution to the equation

$$x - 13x = 3x - 6\,?$$

(a) $\dfrac{2}{5}$ (b) $-\dfrac{1}{5}$ (c) $\dfrac{1}{3}$ (d) $-\dfrac{6}{17}$

2. Further Equations

We are now ready to move on to slightly more sophisticated examples.

Example 2 Find the solution to the equation

$$5(x-3)-7(6-x)=24-3(8-x)-3$$

Solution

Removing the brackets from both sides first and then simplifying:

$$\begin{aligned} 5(x-3)-7(6-x) &= 24-3(8-x)-3 \\ 5x-15-42+7x &= 24-24+3x-3 \\ 5x+7x-15-42 &= 3x-3 \\ 12x-57 &= 3x-3\,. \end{aligned}$$

Adding 57 to both sides:

$$12x = 3x-3+57 = 3x+54$$

Subtracting $3x$ from both sides:

$$12x-3x=54 \quad \text{or} \quad 9x=54 \quad \text{giving} \quad x=6\,.$$

Exercise 2. Find the solution to each of the following equations. (Click on green letters for solutions.)

(a) $2x + 3 = 16 - (2x - 3)$

(b) $8(x - 1) + 17(x - 3) = 4(4x - 9) + 4$

(c) $15(x - 1) + 4(x + 3) = 2(7 + x)$

Quiz Which of the following is the solution to the equation

$$5x - (4x - 7)(3x - 5) = 6 - 3(4x - 9)(x - 1)\,?$$

(a) -2 (b) -1 (c) 2 (d) 4

When fractions occur we can sometimes transform the equation to one that does not involve fractions.

Example 3 Find the solution to the equation

$$(4x/5) - (7/4) = (x/5) + (x/4)\,.$$

Solution

The least common multiple of the denominators in the equation is $4 \times 5 = 20$ and we proceed as follows:

$$\begin{aligned} 20\left(\frac{4x}{5} - \frac{7}{4}\right) &= 20\left(\frac{x}{5} + \frac{x}{4}\right) \\ \frac{20}{1}\cdot\frac{4x}{5} - \frac{20}{1}\cdot\frac{7}{4} &= \frac{20}{1}\cdot\frac{x}{5} + \frac{20}{1}\cdot\frac{x}{4} \\ 16x - 35 &= 4x + 5x \\ 16x - 35 &= 9x\,. \end{aligned}$$

adding 35 to both sides and subtracting $9x$ from both sides leads to

$7x = 35$ so $x = 5$ is the solution to the equation.

Exercise 3. Find the solution to each of the following equations. (Click on green letters for solutions.)

(a) $5x - 6(x - 5) = 2(x + 5) + 5(x - 4)$

(b) $(x + 15)(x - 3) - (x^2 - 6x + 9) = 30 - 15(x - 1)$

(c) $(x - 2)/2 + (x + 10)/9 = 5$

Quiz Which of the following is the solution to the equation

$$(x - 4)/7 = (x - 10)/5\,?$$

(a) 11 (b) -10 (c) 19 (d) 25

3. Quiz on Equations

Begin Quiz In each of the following, solve the equation and choose the solution from the options given.

1. $4(x+2)/5 = 7 + 5x/13$
(a) 5 (b) 13 (c) -5 (d) -13

2. $(x+20)/9 + 3x/7 = 6$
(a) 9 (b) 7 (c) 5 (d) 2

3. $(x+35)/6 - (x+7)/9 = (x+21)/4$
(a) -5 (b) 2 (c) 4 (d) -1

4. $(x+1)(2x+1) = (x+3)(2x+3) - 14$
(a) 1 (b) -1 (c) 2 (d) -2

End Quiz Score: Correct

Solutions to Exercises

Exercise 1(a)
Dividing both sides by 3 gives

$$\frac{3x}{3} = \frac{18}{3}$$

or

$$x = 6\,.$$

Click on green square to return

□

Exercise 1(b)
Dividing both sides by 7 gives

$$\frac{7x}{7} = -\frac{14}{7}$$

or

$$x = -2\,.$$

Click on green square to return

□

Exercise 1(c)
Dividing both sides by -2 gives

$$\frac{-2x}{-2} = \frac{-10}{-2}$$

or

$$x = 5\,.$$

Click on green square to return □

Exercise 1(d)

Here 7 is the highest common factor of 28 and 35. First let us divide both sides by this.

$$\begin{aligned} 28x &= 35 \\ \frac{28x}{7} &= \frac{35}{7} \\ 4x &= 5\,. \\ \text{Now divide } &\text{ both } \text{ sides by } 4\,. \\ \frac{4x}{4} &= \frac{5}{4} \\ x &= \frac{5}{4}\,. \end{aligned}$$

The solution is thus $x = 5/4$.

Click on green square to return □

Exercise 1(e)
First let us simplify both sides. The left hand side is

$$5x - 3x - 12x = 5x - 15x = -10x\,.$$

The right hand side is

$$29 - 2 - 7 = 29 - 9 = 20\,.$$

The original equation is thus

$$-10x = 20$$

and the solution to this is obtained by dividing both sides of the equation by -10.

$$\frac{-10x}{-10} = \frac{20}{-10}\,,$$

so that

$$x = -2\,.$$

Click on green square to return □

Exercise 1(f)
In this case we must *multiply* both sides by 5.

$$\begin{aligned} -\frac{x}{5} &= 3 \\ -\frac{5 \times x}{5} &= 5 \times 3 \\ -x &= 15 \\ x &= -15\,, \end{aligned}$$

and the solution in this case is $x = -15$.

Click on green square to return □

Exercise 2(a)

$$\begin{aligned} 2x+3 &= 16-(2x-3) \\ &= 16-2x+3 \\ &= 19-2x \end{aligned}$$

Now add $2x$ to both sides and subtract 3 from both sides

$$\begin{aligned} 2x+3 &= 19-2x \\ 4x+3 &= 19 \\ 4x &= 19-3 \\ 4x &= 16 \end{aligned}$$

and the solution is $x = 4$. This can be checked by putting $x = 4$ in both sides of the first equation above and noting that each side will have the value 11.

Click on green square to return

Exercise 2(b)

$$\begin{aligned}
8(x-1)+17(x-3) &= 4(4x-9)+4 \\
8x-8+17x-51 &= 16x-36+4 \\
25x-59 &= 16x-32 \\
25x-16x-59 &= -32 \\
9x-59 &= -32 \\
9x &= 59-32 \\
9x &= 27 \\
x &= 3\,.
\end{aligned}$$

Inserting $x = 3$ into the equation we can check that both sides have the value 16.

Click on green square to return

Exercise 2(c)

$$\begin{aligned}
15(x-1)+4(x+3) &= 2(7+x) \\
15x-15+4x+12 &= 14+2x \\
19x-3 &= 2x+14 \\
19x-2x-3 &= 14 \\
17x-3 &= 14 \\
17x &= 14+3=17 \\
x &= 1
\end{aligned}$$

Inserting $x = 1$ into the equation we can check that both sides have the value 16. Click on green square to return □

Exercise 3(a)

$$\begin{aligned} 5x - 6(x-5) &= 2(x+5) + 5(x-4) \\ 5x - 6x + 30 &= 2x + 10 + 5x - 20 \\ -x + 30 &= 7x - 10 \\ 30 &= x + 7x - 10 \\ 30 &= 8x - 10 \\ 30 + 10 &= 8x \\ 8x &= 40 \\ x &= 5 \end{aligned}$$

Click on green square to return □

Exercise 3(b) First, using FOIL, we expand

$$(x+15)(x-3) = x^2 - 3x + 15x - 45 = x^2 + 12x - 45$$

Now we have

$$\begin{aligned}
(x+15)(x-3) - (x^2 - 6x + 9) &= 30 - 15(x-1) \\
x^2 + 12x - 45 - x^2 + 6x - 9 &= 30 - 15x + 15 \\
18x - 54 &= 45 - 15x \\
18x + 15x - 54 &= 45 \\
33x - 54 &= 45 \\
33x &= 45 + 54 \\
33x &= 99 \\
x &= 3
\end{aligned}$$

Click on green square to return □

Exercise 3(c)

This time we multiply both sides by 2×9.

$$\begin{aligned}
\frac{(x-2)}{2} + \frac{(x+10)}{9} &= 5 \\
\frac{2\times 9}{1} \times \frac{(x-2)}{2} + \frac{2\times 9}{1} \times \frac{(x+10)}{9} &= 2 \times 9 \times 5 \\
9(x-2) + 2(x+10) &= 90 \\
9x - 18 + 2x + 20 &= 90 \\
11x + 2 &= 90 \\
11x &= 88 \\
x &= 8
\end{aligned}$$

Click on green square to return □

Solutions to Quizzes

Solution to Quiz:

Simplify both sides first:

$$\begin{aligned} 8x + 5x - 3x &= 13x - 3x \\ &= 10x\,. \end{aligned}$$

$$\begin{aligned} 17 - 9 + 22 &= 8 + 22 \\ &= 30\,. \end{aligned}$$

The equation to be solved is thus $10x = 30$ and this clearly has solution $x = 3$.

End Quiz

Solution to Quiz: l

$$\begin{aligned} x - 13x &= 3x - 6 \\ -12x &= 3x - 6\,. \\ 0 &= 12x + 3x - 6 \\ 15x - 6 &= 0 \\ 15x &= 6 \end{aligned}$$

$$x = \frac{6}{15}$$

$$= \frac{2}{5}\,.$$

End Quiz

Solution to Quiz: First expand the brackets separately using FOIL (see the package on Brackets):

$$\begin{aligned}(4x-7)(3x-5) &= \overset{\mathbf{F}}{12x^2} - \overset{\mathbf{O}}{20x} - \overset{\mathbf{I}}{21x} + \overset{\mathbf{L}}{35} \\ &= 12x^2 - 41x + 35\,. \\ (4x-9)(x-1) &= \overset{\mathbf{F}}{4x^2} - \overset{\mathbf{O}}{4x} - \overset{\mathbf{I}}{9x} + \overset{\mathbf{L}}{9} \\ &= 4x^2 - 13x + 9\,.\end{aligned}$$

These can now be substituted, *carefully,* into the equation:

$$\begin{aligned}5x - [(4x-7)(3x-5)] &= 6 - 3[(4x-9)(x-1)] \\ 5x - [12x^2 - 41x + 35] &= 6 - [4x^2 - 13x + 9] \\ 5x - 12x^2 + 41x - 35 &= 6 - 4x^2 + 13x - 9 \\ -12x^2 + 46x - 35 &= -12x^2 + 39x - 21\,.\end{aligned}$$

Notice the extra pair of *square* brackets in the first equation above. These are to emphasise that the *negative* sign multiplies *all* of the parts inside the [ ]brackets. The procedure now follows in an obvious manner. Add $12x^2$ to both sides, subtract $39x$ from both sides then add 35 to both sides:

$$\begin{aligned} -12x^2 + 46x - 35 &= -12x^2 + 39x - 21 \\ 46x - 35 &= 39x - 21 \\ 46x - 39x - 35 &= -21 \\ 46x - 39x &= 35 - 21 \\ 7x &= 14 \\ x &= 2\,. \end{aligned}$$

End Quiz

Solution to Quiz:
The highest common factor of the denominators is $5 \times 7 = 35$. Multiplying both sides of the equation by this

$$
\begin{aligned}
\frac{35}{1} \times \frac{(x-4)}{7} &= \frac{35}{1} \times \frac{(x-10)}{5} \\
5(x-4) &= 7(x-10) \\
5x - 20 &= 7x - 70 \\
5x - 20 + 70 &= 7x - 70 + 70 \\
5x + 50 &= 7x \\
50 &= 7x - 5x = 2x \\
x &= 25
\end{aligned}
$$

so that $x = 25$ is the solution. This can be checked by putting this value into the original equation and showing that each side will have the value 3.

End Quiz