Matrix Algebra

A matrix is a method of displaying information in a tabular form consisting of rows and columns eg.

	a	b	c
x	2	2	3
y	3	1	2
z	4	3	1

The letter a, b and c could represent three branches of a chain of garages and the letters x, y and z three different models of car on sale at the branches. The matrix would then be indicating the stock of each model at each branch in an easily interpreted way.

This example of a matrix has three rows and three columns so is a 3 x 3 matrix. This is called the matrix order. A 5 x 4 order would therefore have 5 rows and 4 columns.

The elements of a matrix are shown enclosed in brackets, eg matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$

Many arithmetic operations can be carried out on matrices provided certain conditions are met. The conditions vary depending on the operation to be carried out.

Addition and Subtraction of Matrices

Matrices can only be added or subtracted provided they are of the same order.

Eg.1 matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ and matrix $B = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$

To determine A + B, the corresponding elements are added

$\therefore$ $A + B = \begin{pmatrix} 4 & 1 \\ 2 & 6 \end{pmatrix}$

Eg.2 matrix $A = \begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix}$ and matrix $B = \begin{pmatrix} 2 & -2 \\ -1 & 1 \end{pmatrix}$

To determine A – B, the corresponding elements are subtracted

$\therefore$ $A - B = \begin{pmatrix} 1 & 4 \\ 2 & -1 \end{pmatrix}$

Care must be taken with the signs especially when subtracting.

Scalar Multiplication of Matrices

When the matrix is to be multiplied by a single numeral or term, each element is multiplied in turn by the multiplicand

Eg.3 if matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ then $5(A) = \begin{pmatrix} 10 & 5 \\ 5 & 15 \end{pmatrix}$ or $2y(A) = \begin{pmatrix} 4y & 2y \\ 2y & 6y \end{pmatrix}$

Multiplication of Matrices

One matrix may be multiplied by another matrix provided there are as many columns in the first matrix as there are rows in the second matrix.

Eg.4 matrix A = $(1\ \ 2\ \ 3)$ matrix B = $\begin{pmatrix} 3 & 1 & 3 \\ 1 & 2 & 2 \\ 2 & 1 & 1 \end{pmatrix}$

These matrices can be multiplied together to produce the 'product matrix' AB

To multiply the matrices, the elements of the first matrix are multiplied by the elements of the first column of the second matrix and the results totalled.

Eg.5 matrix A = $(1\ \ 2\ \ 3)$ matrix B = $\begin{pmatrix} 3 & 1 & 3 \\ 1 & 2 & 2 \\ 2 & 1 & 1 \end{pmatrix}$

The product matrix AB =
$1 \times 3 + 2 \times 1 + 3 \times 2 = 11$ (1st column total)
$1 \times 1 + 2 \times 2 + 3 \times 1 = 8$ (2nd column total)
$1 \times 3 + 2 \times 2 + 3 \times 1 = 10$ (3rd column total)

$\therefore AB = (11\ \ 8\ \ 10)$

If the multiplying matrix has more than one row, the process is repeated

Eg.6 If matrix A = $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 1 \end{pmatrix}$ and matrix B = $\begin{pmatrix} 3 & 1 \\ 1 & 2 \\ 2 & 2 \end{pmatrix}$ determine product matrix AB

$\therefore AB = 1 \times 3 + 2 \times 1 + 3 \times 2 = 11$ (1st column) 1st row A x 1st column B
$1 \times 1 + 2 \times 2 + 3 \times 2 = 11$ (2nd column) 1st row A x 2nd column B
then $1 \times 3 + 3 \times 1 + 1 \times 2 = 8$ (1st column, 2nd row) 2nd row A x 1st column B
$1 \times 1 + 3 \times 2 + 1 \times 2 = 9$ (2nd column, 2nd row) 2nd row A x 2nd column B

$\therefore AB = \begin{pmatrix} 11 & 11 \\ 8 & 9 \end{pmatrix}$

The product matrix will always have the same number of rows as the multiplying matrix and the same number of columns as the second matrix.

In normal arithmetic and algebraic multiplication $3 \times 2 = 2 \times 3$ or $a \times b = b \times a$
The left hand side and right hand sides of the equations are said to be commutative.

In general, matrix multiplication is non-commutative ie. $AB \neq BA$

Eg.7 if matrix A = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and matrix B = $\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$ then AB = $\begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix}$ and BA = $\begin{pmatrix} 23 & 34 \\ 31 & 46 \end{pmatrix}$

Matrix Algebra Worksheet 1

Express each of the following as a single matrix

1. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 2 & 4 \\ 3 & 1 \end{pmatrix}$

2. $\begin{pmatrix} 6 & 11 \\ 7 & 4 \end{pmatrix} + \begin{pmatrix} 9 & 13 \\ 14 & 2 \end{pmatrix}$

3. $\begin{pmatrix} 9 & 0 \\ 0 & 8 \end{pmatrix} + \begin{pmatrix} 0 & 7 \\ 5 & 0 \end{pmatrix}$

4. $\begin{pmatrix} 11 & -1 \\ -7 & 4 \end{pmatrix} + \begin{pmatrix} -3 & 15 \\ 12 & -6 \end{pmatrix}$

5. $2\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} + 3\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$

6. $3\begin{pmatrix} 6 & 1 \\ -2 & 3 \end{pmatrix} + 5\begin{pmatrix} -4 & 7 \\ 0 & 9 \end{pmatrix}$

7. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

8. $\begin{pmatrix} 3 & 4 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 5 \end{pmatrix}$

Matrix Algebra Worksheet 2

Express each of the following as a single matrix

1. $\begin{pmatrix} 3a & 2a \\ 5a & 6a \end{pmatrix} + \begin{pmatrix} 0 & 9a \\ 4a & 11a \end{pmatrix}$

2. $\begin{pmatrix} -2b & 3b \\ 4b & -7b \end{pmatrix} + \begin{pmatrix} 8b & 2b \\ -6b & -9b \end{pmatrix}$

3. $2\begin{pmatrix} a & b \\ c & d \end{pmatrix} + 3\begin{pmatrix} a & -b \\ -c & d \end{pmatrix}$

4. $p\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} + q\begin{pmatrix} 0 & -a \\ a & 0 \end{pmatrix} + r\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

5. $\begin{pmatrix} a & b \\ b & a \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

6. $\begin{pmatrix} a & 2a \\ -a & 2a \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

7. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

8. $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$