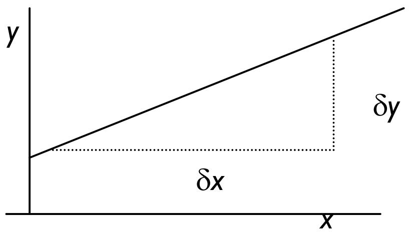
Introduction to Differentiation

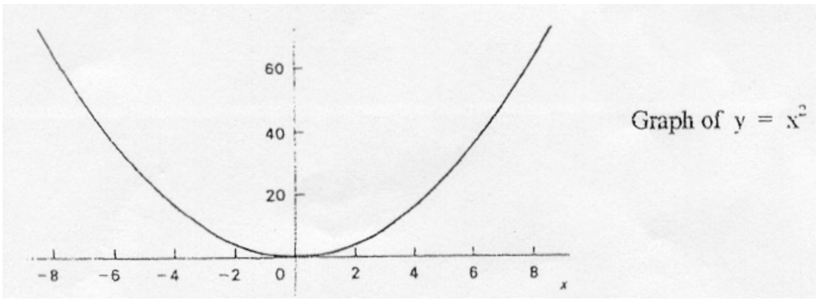
Differentiation is a technique within the topic of Differential Calculus. Differentiation is a method of finding a formula for the slope of a graph at any specified point.
Eg. The slope of a straight line graph is the same at all points, so slope = m, a constant



graph of $y = mx + c$

slope $= \dfrac{\delta y}{\delta x}$

The slope of a curve will vary depending on the point at which the slope is measured



To determine the slope, a tangent must be drawn at the point of interest (usually defined by giving the appropriate value of x i.e. where $x = 2$). The slope of the tangent is then calculated in the same way as for straight line graphs. If the value of the slope is required at many different points, many tangents would be required. This is tedious and is dependent on how accurately the tangents are drawn. Differential calculus gives a method of determining the slope of any curve, at any point, without the need to plot the curve in the first place. The results will also be more accurate than is likely from a graphical method.

Take the curve of $y = x^2$ shown above:
Select two values on the x axis ie x and $(x + h)$. Since $y = x^2$ at all points on the curve then

$y = x^2$ at the first point

$y = (x + h)^2$ at the second point

$$\text{slope} = \frac{\text{difference in the value of } y}{\text{difference in the value of } x}$$

$$= \frac{\text{value of } y \text{ at second point} - \text{value of } y \text{ at first point}}{\text{value of } x \text{ at second point} - \text{value of } x \text{ at first point}}$$

$$= \frac{(x + h)^2 - x^2}{(x + h) - x}$$

$$= \frac{x^2 + 2xh + h^2 - x^2}{x + h - x} = \frac{2xh + h^2}{h} = \frac{h(2x + h)}{h} = 2x + h$$

as the value of h decreases, it becomes negligible compared to the value of $2x$, giving a final value of $2x$ for the slope.

$$\therefore \text{ if } y = x^2 \text{ then } \frac{\delta y}{\delta x} = 2x$$

In similar manner, the following can be determined:

If $y = x^3$ then $\frac{\delta y}{\delta x} = 3x^2$

$y = x^4$ then $\frac{\delta y}{\delta x} = 4x^3$

$y = x^5$ then $\frac{\delta y}{\delta x} = 5x^4$

$y = x^n$ then $\frac{\delta y}{\delta x} = nx^{(n-1)}$

i.e. multiply by the original power and reduce the original power by 1. This process is known as *differentiation* and the result as the *differential coefficient.*

If the term to be differentiated already has a coefficient, it is multiplied by the original power

i.e. if $y = 4x^3$ then $\frac{\delta y}{\delta x} = 12x$

Where terms have x to the power of 1, $\frac{\delta y}{\delta x} =$ the coefficient of the term

i.e. if $y = 5x$ then $\frac{\delta y}{\delta x} = 5$

Terms that are numerical only, become zero on differentiation

i.e. if $y = 7$ then $\frac{\delta y}{\delta x} = 0$

Expressions consisting of multiple terms are differentiated by considering each term separately

i.e. if $y = 4x^3 - 7x^2 + 5x - 7$, $\frac{\delta y}{\delta x} = 12x^2 - 14x + 5$

Expressions must be expanded to give separate terms before differentiation

i.e. if $y = 4x(x^2 + 3x)$, $\frac{\delta y}{\delta x} = 12x^2 + 24x$

$y = 4x^3 + 12x^2$

If the variable is a denominator in any term, it must be converted to a numerator even if it results in a negative power. Rules for differentiating apply in the same way to negative powers

i.e. if $y = 3x^3 + \frac{2}{x^2} - \frac{4}{x} = 3x^3 + 2x^{-2} - 4x^{-1}$

$$\frac{\delta y}{\delta x} = 9x^2 - 4x^{-3} + 4x^{-2}$$

All normal algebraic rules for signs apply so be careful with signs!

Value of the differential coefficient

The differential coefficient of an expression $\left(\frac{\delta y}{\delta x}\right)$ gives the formula for the value of the slope of the curve for the expression at any point. To calculate he numerical value of the slope at any point, the value of x at the point of interest is substituted into the differential coefficient.

e.g. Determine the slope of the expression

$$y = 2x^2 + 3x \quad \text{where} \quad x = 3$$

$$\frac{\delta y}{\delta x} = 4x + 3$$

Value of slope where $x = 3$ $\quad \frac{\delta y}{\delta x} = 4(3) + 3 = 15$

Introduction to Differentiation Worksheet 1

Differentiate the following:

1. $x^2 + 1$
2. $x^3 - 3$
3. $x^4 + 2$
4. $x^5 - 4$
5. $2x^2 - 3$
6. $3x^2 + x$
7. $2x^2 - 3x$
8. $3x^2 + 2x$

Determine the gradient of the following expressions where $x = 2$

9. $y = 3x + 2$
10. $y = x^2 - x$

Introduction to Differentiation Worksheet 2

Differentiate the following:

1. $2x^4 + 5x + 1$
2. $5x^4 - 6x^3 + 7x - 5$
3. $x^3 - x^2 + x + 1$
4. $\frac{1}{2}x^2 - 3x + 4$
5. $\frac{x^3}{3} + \frac{1}{4}x^2$
6. $\frac{2x^3}{3} - \frac{4x^5}{5}$

Determine the gradient of the following expressions where $x = 3$

7. $y = 2x^3 - 4x$
8. $y = x^4 - 3x^3$