Simultaneous Linear Equations

If a linear equation has two unknowns, it is not possible to solve. Many combinations of values for the unknowns might satisfy the equation

Eg. $2x + 4y = 10$ works if $x = 1$ and $y = 2$
$x = -10$ and $y = 5$
$x = 4$ and $y = 0.5$
$x = -7$ and $y = 6$ etc

Without further information it is not possible to say which pair of values is the relevant solution to the equation. However, if the values for the two unknowns must also satisfy another equation, it will be possible to determine the one pair of values that are correct.

Eg. if it is known that $2x + 4y = 10$ and $x + 8y = 17$

Then x must equal 1 and y must equal 2, the other pairs of values that satisfy the first equation do not satisfy the second equation. When the same values satisfy two equations they are said to be 'simultaneous'.

To solve a pair of simultaneous equations

Remember: any action may be carried out on an equation provided the same action is applied to both sides. Note ....(1) and ....(2) are labels for the equations so we can refer to them.

$2x + 4y = 10$ ……….(1)
$x + 8y = 17$ ……….(2)

To proceed with this method we need to make the coefficients of one of the unknowns the same in both equations. This is not so, but multiplying equation (2) by 2 would give;

$2x + 16y = 34$ ……….(3)

Now we subtract equation (3) from equation (1), in effect using another equation to 'do the same to both sides'

$$\begin{array}{rcl} 2x + 4y & = & 10 \quad - \\ \underline{2x + 16y} & \underline{=} & \underline{34} \\ -12y & = & -24 \\ \therefore \quad y & = & \dfrac{-24}{-12} = 2 \end{array}$$

substituting this value for y back into equation (1) gives

$$2x + 4(2) = 10$$
$$\therefore \quad 2x = 10 - 8 = 2$$
$$\therefore \quad x = 1$$

It may be necessary to multiply both original equations by convenient factors to obtain the same coefficient of one of the unknowns in both equations

Eg. solve for *x* and *y* the following pair of simultaneous equations

$3x + 2y = 7$ ……….(1)
$2x - 5y = -8$ ……... (2)

there may be several multiplication factors that could be used to obtain the same coefficient of one of the unknowns in both equations – only personal preference need dictate which to use. In this example equation (1) is multiplied by 2 and equation (2) is multiplied by 3 to give

$6x + 4y = 14$ ………(3)
$6x - 15y = -24$ ……...(4)

subtract equation (4) from equation (3)

$$\begin{array}{rl} 6x + 4y = & 14 \\ \underline{6x - 15y =} & \underline{-24} \\ 19y = & 38 \\ \therefore\ y = & \frac{38}{19} = 2 \end{array}$$

the value of *x* can now be determined by substitution into either of the two original equations or the two derived equations

$$\begin{array}{rl} 3x + 2y = & 7 \\ 3x + 4 = & 7 \\ \therefore\ x = & 1 \end{array}$$

the value of *x* could also be determined using the same method as for *y*

$3x + 2y = 7$ ……….(1) $\times$ 5
$2x - 5y = -8$ ……... (2) $\times$ -2

$15x + 10y = 35$ ………(5)
$-4x + 10y = 16$ ……....(6)

subtract equation (6) from equation (5)

$$\begin{array}{rl} 15x + 10y = & 35 \quad - \\ \underline{-4x + 10y =} & \underline{16} \\ 19x = & 19 \end{array}$$

$$\therefore\ x = \frac{19}{19} = 1$$

the second method is often the easiest method to determine the value for the second unknown if the value of the first includes fractions or decimals.

Simultaneous linear equations - Worksheet 1

Solve the following pairs of simultaneous equations for both unknowns:

1. $x + y = 8, \ x - y = 4$
2. $2x + y = 7, \ 2x - y = 3$
3. $2x + 3y = 5, \ 3x + 4y = 7$
4. $5x + 4y = 22, \ 3x + 5y = 21$

Simultaneous linear equations - Worksheet 2

Solve the following pairs of simultaneous equations for both unknowns:

1. $3(x + 1) = 2(y + 2), \ 3(x + 3) = 4(y + 2)$
2. $4(x + y + 1) = 7(x + 2y), \ 4(2x + y) = 10(x + y + 1)$
3. $3(x + 1) = 2(2y + 1), \ 2(y + 3) = 4(x + 1)$
4. $3(x + y + 2) = 7x, \ 2x = 3y$