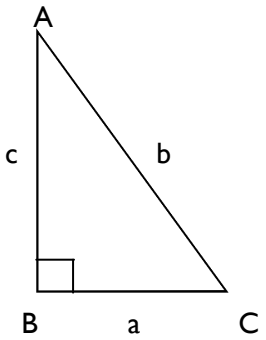
# Introduction to Trigonometry

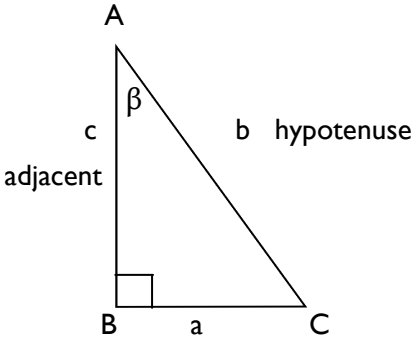
Trigonometry is based on the ratio of sides of right-angled triangles.

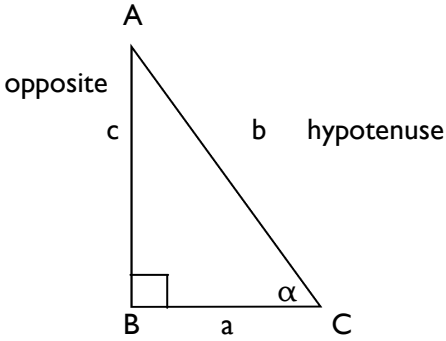
In trigonometry, angles are identified by a single CAPITAL letter; sides are identified by a single lower cased letter, opposite the angle.




i.e. side b is opposite angle B

In trigonometry, each side of a right-angled triangle is given a name.
The hypotenuse is the longest side of the triangle and is always the side opposite the right-angle. The other two sides are the opposite or adjacent. Depending on which angle is being referred to.



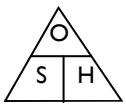




With reference to angle β side a is the opposite and side c is the adjacent.

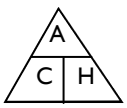
With reference to angle α side a is the adjacent and side c is the opposite.

When certain pairs of sides are used to establish a ratio, they give the basic trigonometrical (trig) ratios of sine, cosine and tangent.

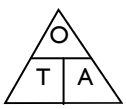
Sine (sin) = $\frac{\text{opposite}}{\text{hypotenuse}}$




Cosine (cos) = $\frac{\text{adjacent}}{\text{hypotenuse}}$




Tangent (tan) = $\frac{\text{opposite}}{\text{adjacent}}$




The correct pairs must be used and they must be in the correct order

i.e. sin = $\frac{\text{opposite}}{\text{hypotenuse}}$ NOT $\frac{\text{hypotenuse}}{\text{opposite}}$

The numerical value of the trig ratios for any angle can be found using tables or a scientific calculator.

Find the relevant trig function button on the calculator and type in the angle. The trig function value will be displayed. E.g. find the value of sin 45°

If the trig function and it value are known, the corresponding angle can be found.

E.g. if the sin of an unknown angle is 0.8660, what size is the angle?
Find the inverse sin or $\sin^{-1}$ button on the calculator (Shift sin) and enter 0.8660 into the calculator. The display will show 60. So, 0.8660 is the sin of 60°. This is usually written as

$\sin^{-1}\ 0.8660 = 60°$

## 'Solve' the triangle

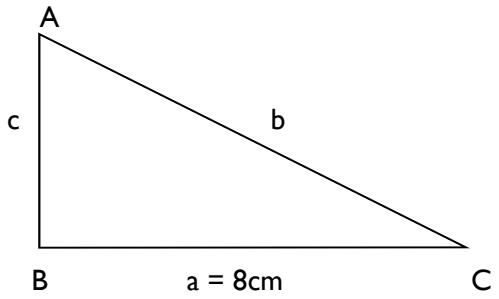
For any right-angled triangle, provided two pieces of information are known, (sides or angles) trig ratios can be used to 'solve' the triangle.

Solve means determine the size of all angles and the length of all sides.

To calculate the length of a side, an angle and one side must already be known.

To calculate the size of an angle, two sides must already be known.

E.g. in the triangle ABC which is right-angled at B, side a = 8cm and angle C = 36°. Solve the triangle




always draw a diagram from the information given

Information already known is one side and one angle – so a side must be calculated.

To find side c (could have chosen b – it doesn't matter which is found first)
Angle C = 36°. Side already known relative to angle C is the adjacent. Side required (c) relative to angle C is the opposite.

Trig function that relates adjacent and opposite is tan

$\therefore \tan C = \frac{\text{opposite}}{\text{adjacent}}$

$\therefore \tan 36° = \frac{c}{8}$

$\therefore 0.7265 = \frac{c}{8}$ $\quad \therefore c = 0.7265 \times 8 = 5.81\text{cm}$

To calculate side b

This time the side required is the hypotenuse, and the side already known is still the adjacent. The trig function that relates adjacent and hypotenuse is cos.

$$\therefore \cos C = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\therefore \cos 36° = \frac{8}{\text{hypotenuse}}$$

$$\therefore 0.8090 = \frac{8}{\text{hyp}} \qquad \therefore \text{hyp} = \frac{8}{0.8090} = 9.89\text{cm}$$

To calculate the size of angle A

All three sides are known, any two can be used to determine an angle.

Relative to angle A, side a is the opposite and side b is the hypotenuse. The trig function that relates opposite and hypotenuse is sin

$$\therefore \sin A = \frac{\text{opposite}}{\text{Hypotenuse}}$$

$$= \frac{8}{9.89} = 0.8089$$

$$\therefore \sin^{-1} 0.8089 = 53.99°$$

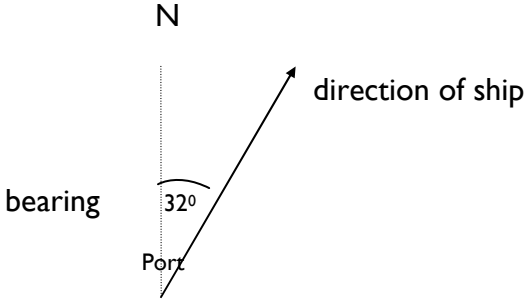
# Practical Problems Using Trigonometry

Once basic trig functions can be used to 'solve' right angled triangles, they can be used to solve practical problems provided right angled triangles can be constructed from the information given. In order to do this, the definition of particular angles must be known. The definitions are given below:

## Bearings

All bearings are quoted from due north, **clockwise**

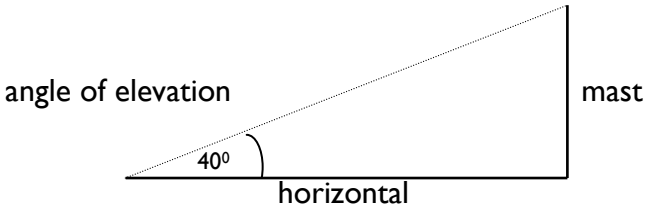
eg. A ship leaves port on a bearing of $32^0$




## Elevation

Angles of elevation are measured from horizontal **upwards**

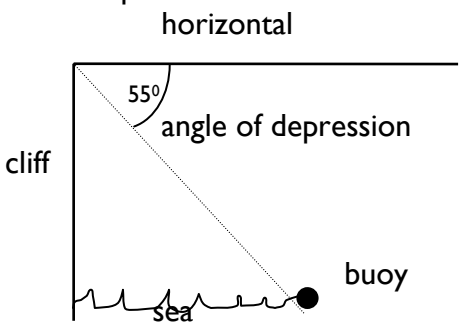
eg. The angle of elevation of the top of a mast is $40^0$




## Depression

Angles of depression are measured from horizontal **downwards**

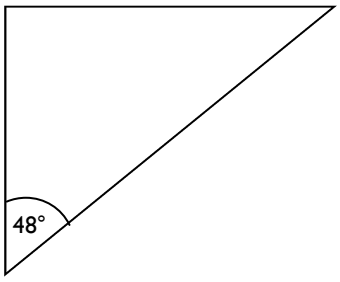
eg. The angle of depression of a buoy at sea from the top of a cliff is $55^0$



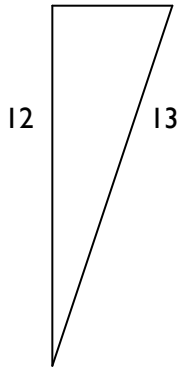
## Introduction to Trigonometry Worksheet 1

Using trig methods solve the following triangles (i.e. all angles and sides)

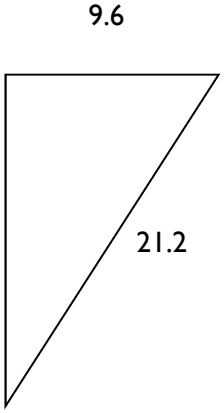
1.




2.




3.




4. The angle of elevation of the top of a tower is $36^0$. If the tower is 25m away from the observer, calculate its height.

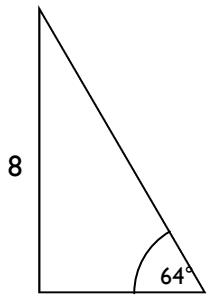
5. A man 1.8m tall is standing 12m away from a tree. If the angle of elevation of the top of the tree is $24^0 14'$, calculate the height of the tree.

6. A point K is 12km due west of a second point L and 25km due south of a third point M. Calculate the bearing of L from M.

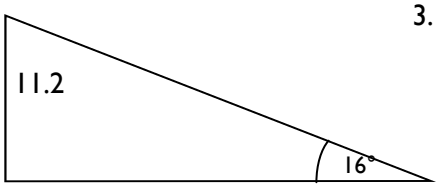
## Introduction to Trigonometry Worksheet 2

Using trig methods solve the following triangles (i.e. all angles and sides)

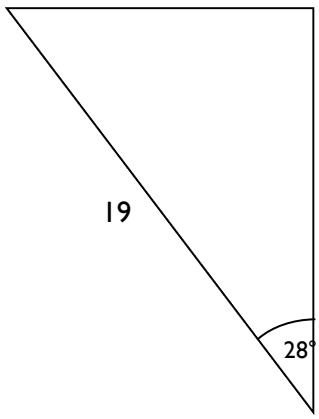
1.




2.




3.




4. Point Y is 1km due north of point X. The bearings of point Z from X and Y are 26°30' and 42°40' respectively. Calculate the distance from point Y to point Z.

5. A ship steams 4km due north of a point then 3km on a bearing of 040°. Calculate the direct distance between the starting and finishing points.

6. The angles of elevation of the top of a tower from A and B, two points in line with its foot are 27° and 32° respectively. If the distance between A and B is 21m, calculate the height of the tower.