Perimeter, Area and Volume of Regular Shapes

Perimeter of Regular Polygons
Perimeter means the total length of all sides, or distance around the edge of a polygon.

For a polygon with straight sides this is the sum of all sides.

Eg. triangle rectangular parallelogram trapezium

\[
\begin{align*}
5 + 5 + 4 &= 14 \text{cm} \\
6 + 6 + 11 + 11 &= 34 \text{cm} \\
8 + 8 + 7 + 3 &= 30 \text{cm} \\
4 + 4 + 9 + 3 &= 20 \text{cm}
\end{align*}
\]
All dimensions given in cm (not drawn to scale)

For polygons with curved sides the perimeter is known as the circumference and is given by the formula

\[
\text{Circumference} = 2\pi r \quad \text{for a circle} \quad \text{and} \quad 2\pi \sqrt{\frac{a^2 + b^2}{2}} \quad \text{for an ellipse}
\]

Where \( \pi \) is a mathematical constant with the value of 3.142 (correct to 3 decimal places)
\( r \) is the radius of the circle (distance from centre to circumference)
\( a \) is the major radius of an ellipse .....
\( b \) is the minor radius of an ellipse .....

Eg. radius = 5

\[
\text{Circumference} = 2\pi r = 2 \times 3.142 \times 5 = 31.42 \text{cm}
\]

Area of Regular Polygons
The area of a polygon is the space it occupies in a single plane.

For squares, rectangles and parallelograms the area is given by

\[
\text{Area} = \text{base} \times \text{height}
\]

Eg.

\[
\begin{align*}
12 \times 12 &= 144 \text{cm}^2 \\
12 \times 4 &= 48 \text{cm}^2 \\
8 \times 7 &= 56 \text{cm}^2
\end{align*}
\]
Height is defined as the perpendicular distance between the pair of parallel sides
All dimensions given in cm (not drawn to scale)

For Triangles  \[ \text{area} = \frac{1}{2} \times \text{base} \times \text{height} \]

Where height is distance from apex to meet base at right angle

\[
\text{Area} = \frac{1}{2} \times 12 \times 3 = 18\text{cm}^2
\]

For Trapeziums  \[ \text{area} = \frac{1}{2} \times \text{sum of parallel sides} \times \text{height} \]

\[
\text{Area} = \frac{1}{2} \times (8 + 14) \times 7 = 77\text{cm}^2
\]

For Circles  \[ \text{area} = \pi r^2 \]

\[
\text{Area} = \pi r^2 \\
= 3.142 \times 5^2 = 78.54\text{cm}^2
\]

For a sector of a Circle  \[ \text{area} = \frac{\text{area of circle} \times \text{sector angle}}{360} \]

\[
\text{Area of sector} = \frac{\pi r^2 \times 60}{360} = 13.1\text{cm}^2
\]

For Ellipse  \[ \text{area} = \pi ab \]

\[
\text{Area} = 3.142 \times 10 \times 5 = 157\text{cm}^2
\]

Complex shapes for which there are no formulas should be divided into simple shapes. The area of each is then calculated and added together to determine the overall area.

\[
\text{Area} = A1 + A2 + A3 + A4 - A5
\]
**Volume of Regular Shapes**

Volume is the amount of space in 3 dimensions occupied by a shape.

**Prism**
A prism is any shape where the cross-sectional area is constant.

For any prism:  \[ \text{Volume} = \text{area of base} \times \text{height} \]

**Rectangular Prism**

- area of base = length \times breadth
- volume = length \times breadth \times height

eg. calculate the volume of a block with a square base of side 6cm and a height of 10cm

\[
\text{volume} = l \times b \times h = 6 \times 6 \times 10 = 360\text{cm}^3
\]

Shaded area is the base

**Triangular Prism**

- area of base = \( \frac{1}{2} \times \text{base} \times \text{height} \)
- volume = \( \frac{1}{2} \times \text{base} \times h_1 \times h_2 \)

eg. determine the volume of a component 16cm long with a triangular cross-section which has a base of 4cm and perpendicular height of 5cm

\[
\begin{align*}
\text{area of base} &= \frac{1}{2} \times 4 \times 5 \\
\text{volume} &= \frac{1}{2} \times 4 \times 5 \times 16 \\
&= 160\text{cm}^3
\end{align*}
\]

Shaded area is the base

**Circular Prism**

- area of base = \( \pi r^2 \)
- volume = \( \pi r^2 \times \text{height} \)

eg. calculate the volume of a cylinder with a radius of 5cm and a height of 4cm.

\[
\begin{align*}
\text{volume} &= \pi r^2 \times \text{height} \\
&= 3.142 \times 5^2 \times 4 = 314.2\text{cm}^3
\end{align*}
\]

Shaded area is the base
The volume of certain non-prismatic shapes can be determined by using the correct formula.

**Sphere**

\[
\text{volume of a sphere} = \frac{4}{3} \pi r^3
\]

eg. determine the volume of a spherical component with the radius of 7cm.

\[
\text{volume} = \frac{4}{3} \times 3.142 \times 7^3 = 1436.76 \text{cm}^3
\]

**Pyramid and cone**

\[
\text{volume} = \frac{1}{3} \times \text{base area} \times \text{height}
\]

**Pyramid**

\[
\text{volume} = \frac{1}{3} \times \text{base area} \times \text{height}
\]

**Cone**

\[
\text{volume} = \frac{1}{3} \pi r^2 \times h
\]

eg. calculate the volume of a cone with base radius of 6cm and perpendicular height of 10cm

\[
\text{Volume} = \frac{1}{3} \times 3.142 \times 6^2 \times 10 = 376.00 \text{cm}^3
\]

Volumes of irregular shapes can be determined by calculation if the mass and density of the material from which it is known or by displacement.

**Calculation of volume using density and mass.**

eg. density of substance from which an irregular object is made is 8500kg/m$^3$. if it has a mass of 425kg, calculate its volume.

\[
\text{Volume} = \frac{\text{mass}}{\text{density}} = \frac{425}{8500} = 0.05 \text{m}^3
\]

**Measurement of volume using displacement**

\[
\begin{array}{c|c|c}
\text{1st reading} & \text{2nd reading} & \text{volume} \\
300cc & 500cc & 2nd reading – 1st reading \\
& & = 500 – 300 = 200cc
\end{array}
\]
Perimeter, Area and Volume of Regular Shapes Worksheet 1

Calculate the area of the following shapes

1. \[ \text{Circle with radius 15 cm} \]
2. \[ \text{Circle with radius 9.5 cm} \]
3. \[ \text{L-Shape with dimensions 15 cm x 15 cm} \]
4. \[ \text{Z-Shape with dimensions 7.8 cm x 4.5 cm} \]
5. \[ \text{T-Shape with dimensions 3.5 cm x 12 cm} \]

6. A water tank is a cuboid with a base of 1.2m by 0.8m. How deep is the water when the tank contains 0.384m³ of water?

7. A classroom is 5m x 6m x 3m. Health regulations require that each student must have a minimum of 5m³ of air. How many students can occupy the room?

Calculate the volume of the following shapes. All dimensions in cm.

8. \[ \text{Rectangular prism with dimensions 2 cm x 2 cm x 12 cm} \]
9. \[ \text{Octahedron with side 2 cm} \]
10. \[ \text{Cylinder with internal radius 0.75 cm and external radius 1.00 cm} \]
Perimeter, Area and Volume of Regular Shapes Worksheet 2

Calculate the shaded area of the following shapes

1. 

2. 

3. 

4. 

5. 

6. An ingot 80 x 10 x 300mm is cast into a cylinder 120mm diameter. Calculate its length.
7. A rivet has a hemispherical head 6mm radius and a stem of 6mm diameter and 15mm length. Calculate the volume of 100 of the rivets.
8. What would be the volume of (a) air (b) plastic in a ball with 25cm diameter made from plastic 2mm thick?

Calculate the volume of the following shapes. All dimensions in cm.

9. 

10. 

11. 

12.