Law of Indices (Ref: Croft & Davison Ch.7)

Terms that have the same base but different powers are different terms and they cannot be added or subtracted.

e.g. $2a^2$ cannot be combined with $3a^3$ by adding or subtracting but provided they have the same BASE they can be multiplied or divided.

e.g. $a^2 \times a^3 = a^5$

Remember the power or index only tells you how many times a number or letter is to be multiplied by itself

So a^2 really means $a \times a$ and a^3 really means $a \times a \times a$,

$a^2 \times a^3$ really means $a \times a \times a \times a \times a$ so $a^2 \times a^3 = a \times a \times a \times a \times a = a^5$

But the power of 5 in the answer to $a^2 \times a^3$ can be found by adding the powers of the two terms. This is called the LAW of INDICES

Multiplication

Law of Indices states that provided the bases are the same, algebraic terms can be multiplied by adding the Indices.

e.g. $x^2 \times x^4 = x^{(2+4)} = x^6$

If the terms to be multiplied have coefficients, they are multiplied as normal

e.g. $3x^2 \times 4x^3 = 12x^{(2+3)} = 12x^5$

Remember the rules covering directed numbers

e.g. $5x^5 \times 2x^{-3} = 10x^{(5+-3)} = 10x^2$

If the bases are not the same, only the coefficients can be combined

e.g. $3a^2 \times 2b^2 = 6a^2b^2$

Division

Law of Indices states that provided the bases are the same, algebraic terms can be divided by subtracting the indices.

eg, $x^5 \div x^2 = x^{(5-2)} = x^3$

Coefficients of terms are divided as normal

eg. $24x^4 \div 6x^2 = 4x^{(4-2)} = 4x^2$

Care must be taken if the terms have NEGATIVE indices

eg. $x^4 \div x^{-3} = x^{(4--3)} = x^7$

Raising to a power

Sometimes terms that already include a power or index must be raised to a further power, in this case the two powers are multiplied

e.g. $(a^2)^3$ This means a^2 must be multiplied by itself 3 times $a^2 \times a^2 \times a^2 = a^{(2\times 3)} = a^6$

If the term has a coefficient it must be raised to the power

e.g. $(3x^2)^4 = 81x^8$ or $(2x^{-3})^5 = 32x^{-15}$

Lowering to a root

The term $36x^2$ is an example of an exact square - this means that it is something multiplied by itself, in this case $6a$ because $6a \times 6a = 36a^2$. So $6a$ is said to be the square root of $36a^2$

The cube root of $27a^3$ would be $3a$ because $3a \times 3a \times 3a = 27a^3$. Finding the root of a number is called lowering the number to a specified root.

To lower a term to a root the power of the term is divided by the root required

e.g. $^2\sqrt{x^6} = x^{(6\div 2)} = x^3$

If the term has a coefficient the root must be found for this as well as for the symbol.

e.g. $^3\sqrt{125x^{12}} = 5x^{(12\div 3)} = 5x^4$ or $^5\sqrt{32b^{20}} = 2b^4$

Fractional and Negative Powers

Fractional powers are another way of expressing roots eg $x^{1/4}$ is another way of saying the fourth root of x or $a^{1/5}$ is the same as the fifth root of a.

If an algebraic term has a negative index, it can be converted into a positive index by taking the reciprocal of the term and changing the sign if the index e.g. $x^{-2} = \frac{1}{x^2}$ or $\frac{5}{x^{-3}} = 5x^3$

Care must be taken in identifying the elements of a term that any power applies to.

e.g. in $5x^3$ the index applies to the x element only

in $(5x)^3$ the index applies to the 5 and the x elements

This is important when taking reciprocals and changing the sign of the index.

e.g. $4x^{-2} = \frac{4}{x^2}$ the negative power only applies to the x element so only this moves to the denominator when the sign of the negative index is changed to a positive value.

Law of indices - Worksheet 1

Simplify the following:

1. $x^2 \times x^4 =$ **2.** $a^3 \times x^5 =$

3. $p^{-3} \times p^2 =$ **4.** $y^{-3} \times y^{-4} =$

5. $a^5 \times a^4 \times a^{-3} =$ **6.** $p^{-3} \times p^{-2} \times p^4 =$

7. $x^2 \times x^{-3} \times x^4 \times x^{-2} =$ **8.** $a^{-2} \times a^{-4} \times a^{-2} \times a^5 =$

Law of indices - Worksheet 2

Simplify the following:

1. $9x^5 \div 3x^2 =$ **2.** $8x^3 \div 2x^{-5} =$

3. $y^{-3} \times y^{-2} \div y^3 =$ **4.** $x^{-4} \times x^{-3} \div x^{-5} =$

5. $(x^3)^2 =$ **6.** $(y^5)^3 =$

7. $(2a^3)^4 =$ **8.** $(3y^{-2})^{-2} =$