Introduction to algebra (Ref: Croft & Davison Ch.6)

Algebra uses symbols or letters as well as figures so that a general formula can be adapted to suite a range of specific sets of values.

An algebraic term can consist of three distinct parts for example: $5x^3$

a) a numerical coefficient or multiplication factor, i.e. 5 in the example above
b) a base or variable,usually a letter but could be a symbol, i.e. *x* in the example above
c) a power or index, i.e. ^{3} in the example above

The power (or index) denotes the number of times the base is multiplied by itself
i.e. a^4 means a x a x a x a

General statements can be converted into the equivalent algebraic statements

e.g. general statement = six times a number plus 4 times another number

algebraic statement $= 6x + 4y$
$= 6a + 4b$
$= 6\alpha + 4\beta$

The x (or a or α) represents the first number and y (or b or β) represents the second number. Any combination of letters or symbols may be used provided a different one is used for each different number. When appropriate values are assigned to each letter, the expression can be evaluated

e.g. if $x = 2$ and $y = 5$, then $6x + 4y = 12 + 20 = 32$
if $x = 4$ and $y = 1$, then $6x + 4y = 24 + 4 = 28$ (any values can be assigned)

Combination of algebraic terms

Provided both BASE and POWER are the same, the terms can be added or subtracted like ordinary numbers

e.g. $3x + 5x - 2x = 6x$ or $5y - 3y + 7y - 11y = -2y$
$4x^2 + 6x^2 = 10x^2$ or $16a^3 - 5a^3 + 3a^3 - 9a^3 = 5a^3$

Numbers where both BASE and POWER are the same are called '**like terms**'

BUT

$3a + 4b - 2c = 3a + 4b - 2c$

the **bases** are different so the terms cannot be combined by addition or subtraction

$3x^2 + 5x^3 = 3x^2 + 5x^3$

this time, although the base (*x*) is the same for both terms, the **powers** are different so again the terms cannot be combined by addition or subtraction. In algebra it is essential to understand that terms with the same base but different powers are NOT like terms.

Algebraic terms are small 'units' of algebraic expressions. They are separated from other terms, before or after, by either a plus (+) sign or a minus (-) sign.
So the algebraic expression $3x^2 + 5x^3$ consists of two separate terms. The expressions $12abc$ and $6x^2y$ both consist of a single term as the different letters are all multiplied together. In algebra, if letters or symbols are written next to each other without an operation sign (+ - x ÷) between them, it means they should be multiplied. It is conventional for letters multiplied together to be arranged in alphabetical order

e.g. $12bdac$ is mathematically the same as, and would be written as, $12abcd$.

It is important to obey this convention otherwise 'like terms' may not be recognised as such and so not be combined to simplify an expression

e.g. $3xyz - 5yxz + 8zyx - 2xzy + 3zxy = 7xyz$

The convention for terms with different powers is – alphabetical followed by descending power order

e.g. $3x + 2xy - 5x^2 - 3x^3 + x^4 + y^3 - 2y^2 = x^4 - 3x^3 - 5x^2 + 3x + 2xy + y^3 - 2y^2$

Notice that the $2xy$ term comes before the y^3 term, even though the power of y is less than that in the y^3 term, this is because it contains the letter x.

Introduction to algebra - Worksheet 1

If the value of $a = 2$, $b = 3$ and $c = 4$, evaluate the following:

1. a^2
2. ab^3
3. $2a^2c$
4. $5a^2 + 6b^2$
5. $a^2 + c^2$
6. $\dfrac{3a^4}{c^2}$
7. $\dfrac{c^5}{ab^3}$
8. $\dfrac{6a^3 + 4b^2}{2c^2}$

Introduction to algebra - Worksheet 2

Simplify the following where appropriate:

1. $3x + 4x^2 - 2x + x^3$
2. $5a^3 - 2a^3 + 6a$
3. $3a + 4b - 3c$
4. $4a + 3a^2 - 2a^2$
5. $2b^3 - 4b^3 + 2b^3$
6. $4ab + 2ba - 5ba$
7. $3xy + 5yx - 2xy^3$
8. $7abc - 3bca + 5bac$
9. $4ab - 2a^2b + 2ab^2$